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Optimal Power Flow

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An Optimal Power Flow (OPF) function schedules the power system controls to optimize an objective function while satisfying a set of nonlinear equality and inequality constraints. The equality constraints are the conventional power flow equations; the inequality constraints are the limits on the control and operating variables of the system. Mathematically, the OPF can be formulated as a constrained nonlinear optimization problem. This section reviews features of the problem and some of its variants as well as requirements for online implementation.

Optimal scheduling of the operations of electric power systems is a major activity, which turns out to be a large-scale problem when the constraints of the electric network are taken into account. This document deals with recent developments in the area emphasizing optimal power flow formulation and deals with conventional optimal power flow (OPF), accounting for the dependence of the power demand on voltages in the system, and requirements for online implementation.

The OPF problem was defined in the early 1960s (Burchett et al., Feb. 1982) as an extension of conventional economic dispatch to determine the optimal settings for control variables in a power network respecting various constraints. OPF is a static constrained nonlinear optimization problem, whose development has closely followed advances in numerical optimization techniques and computer technology. It has since been generalized to include many other problems. Optimization of the electric system with losses represented by the power flow equations was introduced in the 1960s (Carpentier, 1962; Dommel and Tinney, Oct. 1968). Since then, significant effort has been spent on achieving faster and robust solution methods that are suited for online implementation, operating practice, and security requirements.

OPF seeks to optimize a certain objective, subject to the network power flow constraints and system and equipment operating limits. Today, any problem that involves the determination of the instantaneous "optimal" steady state of an electric power system is referred to as an Optimal Power Flow problem. The optimal steady state is attained by adjusting the available controls to minimize an objective function subject to specified operating and security requirements. Different classes of OPF problems, designed for special-purpose applications, are created by selecting different functions to be minimized, different sets of controls, and different sets of constraints. All these classes of the OPF problem are subsets of the general problem. Historically, different solution approaches have been developed to solve these different classes of OPF. Commercially available OPF software can solve very large and complex formulations in a relatively short time, but may still be incapable of dealing with online implementation requirements.

There are many possible objectives for an OPF. Some commonly implemented objectives are:

- fuel or active power cost optimization,
- active power loss minimization,
- minimum control-shift,
- minimum voltage deviations from unity, and
- minimum number of controls rescheduled.

In fuel cost minimization, the outputs of all generators, their voltages, LTC transformer taps and LTC phase shifter angles, and switched capacitors and reactors are control variables. The active power losses can be minimized in at least two ways (Happ and Vierath, July, 1986). In both methods, all the above variables are adjusted except for the active power generation. In one method, the active power generation at the swing bus is minimized while keeping all other generation constant at prespecified values. This effectively minimizes the total active power losses. In another method, an actual expression for the losses is minimized, thus allowing the exclusion of lines in areas not optimized.

The behavior of the OPF solutions during contingencies was a major concern, and as a result, security constrained optimal power flow was introduced in the early 1970s. Subsequently, online implementations became a new thrust in order to meet the challenges of new deregulated operating environments.

20.1 Conventional Optimal Economic Scheduling

Conventional optimal economic scheduling minimizes the total fuel cost of thermal generation, which may be approximated by a variety of expressions such as linear or quadratic functions of the active power generation of the unit. The total active power generation in the system must equal the load plus the active transmission losses, which can be expressed by the celebrated Kron's loss formula. Reserve constraints may be modeled depending on system requirements. Area and system spinning, supplemental, emergency, or other types of reserve requirements involve functional inequality constraints. The forms of the functions used depend on the type of reserve modeled. A linear form is evidently most attractive from a solution method point of view. However, for thermal units, the spinning reserve model is nonlinear due to the limit on a unit's maximum reserve contribution. Additional constraints may be modeled, such as area interchange constraints used to model network transmission capacity limitations. This is usually represented as a constraint on the net interchange of each area with the rest of the system (i.e., in terms of limits on the difference between area total generation and load).

The objective function is augmented by the constraints using a Lagrange-type multiplier lambda, λ. The optimality conditions are made up of two sets. The first is the problem constraints. The second set is based on variational arguments giving for each thermal unit:

$$\frac{\partial F_i}{\partial P_i} = \lambda \left[1 - \frac{\partial P_L}{\partial P_i} \right] \quad \mathrm{i} = 1, \ldots, \mathrm{N} \tag{20.1}$$

The optimality conditions along with the physical constraints are a set of nonlinear equations that requires iterative methods to solve. Newton's method has been widely accepted in the power industry as a powerful tool to solve problems such as the load flow and optimal load flow. This is due to its reliable and fast convergence, known to be quadratic.

A solution can usually be obtained within a few iterations, provided that a reasonably good initial estimate of the solution is available. It is therefore appropriate to employ this method to solve the present problem.

20.2 Conventional OPF Formulation

The optimal power flow is a constrained optimization problem requiring the minimization of:

$$\boldsymbol{f} = (\boldsymbol{x},\boldsymbol{u}) \tag{20.2}$$

subject to

$$\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u}) = 0 \tag{20.3}$$

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{u}) \leq 0 \tag{20.4}$$

$$\boldsymbol{u}^{\min} \leq \boldsymbol{u} \leq \boldsymbol{u}^{\max} \tag{20.5}$$

$$\boldsymbol{x}^{\min} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\max} \tag{20.6}$$

Here **f**(**x**,**u**) is the scalar objective function, **g**(**x**,**u**) represents nonlinear equality constraints (power flow equations), and **h**(**x**,**u**) is the nonlinear inequality constraint of vector arguments **x** and **u**. The vector **x** contains dependent variables consisting of bus voltage magnitudes and phase angles, as well as the MVAr output of generators designated for bus voltage control and fixed parameters such as the reference bus angle, noncontrolled generator MW and MVAr outputs, noncontrolled MW and MVAr loads, fixed bus voltages, line parameters, etc. The vector **u** consists of control variables including:

- real and reactive power generation
- phase-shifter angles
- net interchange
- load MW and MVAr (load shedding)
- DC transmission line flows
- control voltage settings
- LTC transformer tap settings

Examples of equality and inequality constraints are:

- limits on all control variables
- power flow equations
- generation/load balance
- branch flow limits (MW, MVAr, MVA)
- bus voltage limits
- active/reactive reserve limits
- generator MVAr limits
- corridor (transmission interface) limits

The power system consists of a total of N buses, N_G of which are generator buses. M buses are voltage controlled, including both generator buses and buses at which the voltages are to be held constant. The voltages at the remaining (N − M) buses (load buses), must be found.

The network equality constraints are represented by the load flow equations:

$$P_i(V,\delta) - P_{gi} + P_{di} = 0 \tag{20.7}$$

$$Q_i(V,\delta) - Q_{gi} + Q_{di} = 0 \tag{20.8}$$

Two different formulation versions can be considered.

(a) *Polar Form*:

$$P_i(V,\delta) = |V_i| \sum_{1}^{N} |V_i| |Y_{ij}| \cos(\delta_i - \delta_j - \phi_{ij}) \tag{20.9}$$

$$Q_i(V,\delta) = |V_i| \sum_{1}^{N} |V_j| |Y_{ij}| \sin(\delta_i - \delta_j - \phi_{ij}) \tag{20.10}$$

$$Y_{ij} = |Y_{ij}| / \underline{\varphi_{ij}} \tag{20.11}$$

where

P_i = Active power injection into bus i.
Q_i = Reactive power injection into bus i.
$|\tilde{V}_i|$ = Voltage magnitude of bus i.
δ_i = Angle at bus i.
$|\tilde{Y}_{ij}|, \varphi_{ij}$ = Magnitude and angle of the admittance matrix.
P_{di}, Q_{di} = Active and reactive load on bus i.

(b) *Rectangular Form*:

$$P_i(e,f) = e_i \left[\sum_{1}^{N} \left(G_{ij} e_j - B_{ij} f_j \right) \right] + f_i \left[\sum_{1}^{N} \left(G_{ij} f_j + B_{ij} e_j \right) \right] \tag{20.12}$$

$$Q_i(e,f) = f_i \left[\sum_{1}^{N} \left(G_{ij} e_j - B_{ij} f_j \right) \right] - e_i \left[\sum_{1}^{N} \left(G_{ij} f_j + B_{ij} e_j \right) \right] \tag{20.13}$$

e_i = Real part of complex voltage at bus i.
f_i = Imaginary part of the complex voltage at bus i.
G_{ij} = Real part of the complex admittance matrix.
B_{ij} = Imaginary part of the complex admittance matrix.

The control variables vary according to the objective being minimized. For fuel cost minimization, they are usually the generator voltage magnitudes, generator active powers, and transformer tap ratios. The dependent variables are the voltage magnitudes at load buses, phase angles, and reactive generations.

20.2.1 Application of Optimization Methods to OPF

Various optimization methods have been proposed to solve the optimal power flow problem, some of which are refinements on earlier methods. These include:

1. Generalized Reduced Gradient (GRG) method.
2. Reduced gradient method.

3. Conjugate gradient methods.
4. Hessian-based method.
5. Newton's method.
6. Linear programming methods.
7. Quadratic programming methods.
8. Interior point methods.

Some of these techniques have spawned production OPF programs that have achieved a fair level of maturity and have overcome some of the earlier limitations in terms of flexibility, reliability, and performance requirements.

20.2.1.1 Generalized Reduced Gradient Method

The Generalized Reduced Gradient method (GRG), due to Abadie and Carpentier (1969), is an extension of the Wolfe's reduced gradient method (Wolfe, 1967) to the case of nonlinear constraints. Peschon in 1971 and Carpentier in 1973 used this method for OPF. Others have used this method to solve the optimal power flow problem since then (Lindqvist et al., 1984; Yu et al., 1986).

20.2.1.2 Reduced Gradient Method

A reduced gradient method was used by Dommel and Tinney (1968). An augmented Lagrangian function is formed. The negative of the gradient $\partial L/\partial u$ is the direction of steepest descent. The method of reduced gradient moves along this direction from one feasible point to another with a lower value of f, until the solution does not improve any further. At this point an optimum is found, if the Kuhn-Tucker conditions (1951) are satisfied. Dommel and Tinney used Newton's method to solve the power flow equations.

20.2.1.3 Conjugate Gradient Method

In 1982, Burchett et al. used a conjugate gradient method, which is an improvement on the reduced gradient method. Instead of using the negative gradient ∇f as the direction of steepest descent, the descent directions at adjacent points are linearly combined in a recursive manner.

$$\Gamma_k = -\nabla f + \beta_k \Gamma_{k-1} \quad \beta_0 = 0 \tag{20.14}$$

Here, r_k is the descent direction at iteration "k."

Two popular methods for defining the scalar value β_k are the Fletcher-Reeves method (Carpentier, June 1973) and the Polak-Ribiere method (1969).

20.2.1.4 Hessian-Based Methods

Sasson (Oct. 1969) discusses methods (Fiacco and McCormick, 1964; Lootsma, 1967; Zangwill, 1967) that transform the constrained optimization problem into a sequence of unconstrained problems. He uses a transformation introduced by Powell and Fletcher (1963). Here, the Hessian matrix is not evaluated directly. Instead, it is built indirectly starting initially with the identity matrix so that at the optimum point it becomes the Hessian itself.

Due to drawbacks of the Fletcher-Powell method, Sasson et al. (1973) developed a Hessian load flow with an extension to OPF. Here, the Hessian is evaluated and solved unlike in the previous method. The objective function is transformed as before to an unconstrained objective. An unconstrained objective is formed. All equality constraints and only the violating inequality constraints are included. The sparse nature of the Hessian is used to reduce storage and computation time.

20.2.1.5 Newton OPF

Newton OPF has been formulated by Sun et al. (1984), and later by Maria et al. (Aug. 1987). An augmented Lagrangian is first formed. The set of first derivatives of the augmented objective with respect to the control variables gives a set of nonlinear equations as in the Dommel and Tinney method. Unlike in the Dommel and Tinney method where only a part of these are solved by the N-R method, here, all equations are solved simultaneously by the N-R method.

The method itself is quite straightforward. It is the method of identifying binding inequality constraints that challenged most researchers. Sun et al. use a multiply enforced, zig-zagging guarded technique for some of the inequalities, together with penalty factors for some others. Maria et al. used an LP-based technique to identify the binding inequality set. Another approach is to use purely penalty factors. Once the binding inequality set is known, the N-R method converges in a very few iterations.

20.2.1.6 Linear Programming-Based Methods

LP methods use a linear or piecewise-linear cost function. The dual simplex method is used in some applications (Bentall, 1968; Shen and Laughton, Nov. 1970; Stott and Hobson, Sept./Oct. 1978; Wells, 1968). The network power flow constraints are linearized by neglecting the losses and the reactive powers, to obtain the DC load flow equations. Merlin (1972) uses a successive linearization technique and repeated application of the dual simplex method.

Due to linearization, these methods have a very high speed of solution, and high reliability in the sense that an optimal solution can be obtained for most situations. However, one drawback is the inaccuracies of the linearized problem. Another drawback for loss minimization is that the loss linearization is not accurate.

20.2.1.7 Quadratic Programming Methods

In these methods, instead of solving the original problem, a sequence of quadratic problems that converge to the optimal solution of the original problem are solved. Burchett et al. use a sparse implementation of this method. The original problem is redefined as simply, to minimize,

$$f(x) \tag{20.15}$$

subject to:

$$g(x) = 0 \tag{20.16}$$

The problem is to minimize

$$g^T\ p + \frac{1}{2} p^T\ Hp \tag{20.17}$$

subject to:

$$Jp = 0 \tag{20.18}$$

where

$$p = x - x_k \tag{20.19}$$

Here, g is the gradient vector of the original objective function with respect to the set of variables "x." "J" is the Jacobian matrix that contains the derivatives of the original equality constraints with respect to the variables, and "H" is the Hessian containing the second derivatives of the objective function and a linear combination of the constraints with respect to the variables. x_k is the current point of linearization. The method is capable of handling problems with infeasible starting points and can also handle ill-conditioning due to poor R/X ratios. This method was later extended by El-Kady et al. (May 1986) in a study for the Ontario Hydro System for online voltage/var control. A nonsparse implementation of the problem was made by Glavitsch (Dec. 1983) and Contaxis (May, 1986).

20.2.1.8 Interior Point Methods

The projective scaling algorithm for linear programming proposed by N. Karmarkar is characterized by significant speed advantages for large problems reported to be as much as 50:1 when compared to the simplex method (Karmarkar, 1984). This method has a polynomial bound on worst-case running time that is better than the ellipsoid algorithms. Karmarkar's algorithm is significantly different from Dantzig's simplex method. Karmarkar's interior point rarely visits too many extreme points before an optimal point is found. The IP method stays in the interior of the polytope and tries to position a current solution as the "center of the universe" in finding a better direction for the next move. By properly choosing the step lengths, an optimal solution is achieved after a number of iterations. Although this IP approach requires more computational time in finding a moving direction than the traditional simplex method, better moving direction is achieved resulting in less iterations. Therefore, the IP approach has become a major rival of the simplex method and has attracted attention in the optimization community. Several variants of interior points have been proposed and successfully applied to optimal power flow (Momoh, 1992; Vargas et al., 1993; Yan and Quintana, 1999).

20.3 OPF Incorporating Load Models

20.3.1 Load Modeling

The area of power systems load modeling has been well explored in the last two decades of the twentieth century. Most of the work done in this area has dealt with issues in stability of the power system. Load modeling for use in power flow studies has been treated in a few cases (Concordia and Ihara, 1982; IEEE Committee Report, 1973; IEEE Working Group Report, 1996; Iliceto et al., 1972; Vaahedi et al., 1987). In stability studies, frequency and time are variables of interest, unlike in power flow and some OPF studies. Hence, load models for use in stability studies should account for any load variations with frequency and time as well. These types of load models are normally referred to as dynamic load models. In power flow, OPF studies neglecting contingencies, and security-constrained OPF studies using preventive control, time, and frequency, are not considered as variables. Hence, load models for this type of study need not account for time and frequency. These load models are static load models.

In security-constrained OPF studies using corrective control, the time allowed for certain control actions is included in the formulation. However, this time merely establishes the maximum allowable correction, and any dynamic behavior of loads will usually end before any control actions even begin to function. Hence, static load models can be used even in this type of formulation.

20.3.2 Static Load Models

Several forms of static load models have been proposed in the literature, from which the exponential and quadratic models are most commonly used. The exponential form is expressed as:

$$P_m = a_p V^{b_b} \tag{20.20}$$

$$Q_m = a_q V^{bq} \tag{20.21}$$

The values of the coefficients a_p and a_q can be taken as the specified active and reactive powers at that bus, provided the specified power demand values are known to occur at a voltage of 1.0 per unit, measured at the network side of the distribution transformer. A typical measured value of the demand and the network side voltage is sufficient to determine approximately the values of the coefficients, provided the exponents are known. The range of values reported for the exponents vary in the literature, but typical values are 1.5 and 2.0 for b_p and b_q, respectively.

20.3.3 Conventional OPF Studies Including Load Models

Incorporation of load models in OPF studies has been considered in a couple of cases (El-Din et al., 1989; Vaahedi and El-Din, May 1989) for the Ontario Hydro energy management system. In both cases, loss minimization was considered to be the objective. It is concluded by Vaahedi and El-Din (1989) that the modeling of ULTC operation and load characteristics is important in OPF calculations.

The effects of load modeling in OPF studies have been considered for the case where the generator bus voltages are held at prespecified values (Dias and El-Hawary, 1989). Since the swing bus voltage is held fixed at all times (and also the generator bus voltages in the absence of reactive power limit violations), the average system voltage is maintained in most cases. Thus, an increase in fuel cost due to load modeling was noticed for many systems that had a few (or zero) reactive limit violations, and a decrease for those with a noticeable number of reactive limit violations. Holding the generator bus voltages at specified values restricts the available degrees of freedom for OPF and makes the solution less optimal.

Incorporation of load models in OPF studies minimizing fuel cost (with all voltages free to vary within bounds) can give significantly different results when compared with standard OPF results. The reason for this is that the fuel cost can now be reduced by lowering the voltage at the modeled buses along with all other voltages wherever possible. The reduction of the voltages at the modeled buses lowers the power demand of the modeled loads and will thus give the lower fuel cost. When a large number of loads are modeled, the total fuel cost may be lower than the standard OPF. However, a lowering of the fuel cost via a lowering of the power demand may not be desirable under normal circumstances, as this will automatically decrease the total revenue of the operation. This can also give rise to a lower net revenue if the decrease in the total revenue is greater than the decrease in the fuel cost. This is even more undesirable. What is needed is an OPF solution that does not decrease the total power demand in order to achieve a minimum fuel cost. The standard OPF solution satisfies this criterion. However, given a fair number of loads that are fed by fixed tap transformers, the standard OPF solution can be significantly different from the practically observed version of this solution.

Before attempting to find an OPF solution incorporating load models that satisfies the required criterion, we deal with the reason for the problem. In a standard OPF formulation, the total revenue is constant and independent of the solution. Hence, we can define net revenue R_N, which is linearly related to the total fuel cost F_C by the formula:

$$\boldsymbol{R_N = -F_C + constant} \tag{20.22}$$

The constant term is the total revenue dependent on the total power demand and the unit price of electricity charged to the customers. From this relationship we see that a solution with minimum fuel cost will automatically give maximum net revenue. Now, when load models are incorporated at some buses, the total power demand is not a constant, and hence the total revenue will also not be constant. As a result,

$$\boldsymbol{R_N = -F_C + R_T} \tag{20.23}$$

where "R_T" is the total demand revenue and is no longer a constant.

If instead of minimizing the fuel cost, we now maximize the net revenue, we will definitely avoid the difficulties encountered earlier. This is equivalent to minimizing the difference between the fuel cost and the total revenue. Hence we see that, in the standard OPF, the required maximum net revenue is implied, and the equivalent minimum fuel cost is the only function that enters the computations.

20.3.4 Security Constrained OPF Including Load Models

A conventional OPF result can have optimal but insecure states during certain contingencies. This can be avoided by using a security constrained OPF. Unlike in the former, for a security constrained OPF, we can incorporate load models in a variety of ways. For example, we can consider the loads as independent of voltage for the intact system, but dependent on the voltage during contingencies. This can be justified by saying that the voltage deviations encountered during a standard OPF and modeled OPF are small compared to those that can be encountered during contingencies. Since the total power demand for the intact system is not changed, fuel cost comparisons between this case and a standard SCOPF seem more reasonable. We can also incorporate load models for the intact system as well as during contingencies, while minimizing the fuel cost. However, we then encounter the problem discussed in the previous section regarding net earnings. Another approach is to incorporate load models for the intact case as well as during contingencies, while minimizing the total fuel cost minus the total revenue.

20.3.5 Inaccuracies of Standard OPF Solutions

It was stated earlier that the standard OPF (or standard security constrained OPF) solution can give results not compatible with practical observations (i.e., using the control variable values from these solutions) when a fair number of loads are fed by fixed tap transformers. The discrepancies between the simulated and observed results will be due to discrepancies between the voltage at a bus feeding a load through a fixed tap transformer, and the voltage at which the specified power demand for that load occurs. The observed results can be simulated approximately by performing a power flow incorporating load models. The effects of load modeling in power flow studies have been treated in a few cases (Dias and El-Hawary, 1990; El-Hawary and Dias, Jan. 1987; El-Hawary and Dias, 1987; El-Hawary and Dias, July 1987). In all these studies, the specified power demand of the modeled loads was assumed to occur at a bus voltage of 1.0 per unit. The simulated modeled power flow solution will be same as the practically observed version only when exact model parameters are utilized.

20.4 SCOPF Including Load Modeling

Security constrained optimal power flow (abbreviated SCOPF) takes into account outages of certain transmission lines or equipment (Alsac and Stott, May/June 1974; Schnyder and Glavitsch, 1987). Due to the computational complexity of the problem, more work has been devoted to obtaining faster solutions requiring less storage, and practically no attention has been paid to incorporating load models in the formulations. A SCOPF solution is secure for all credible contingencies or can be made secure by corrective means. In a secure system (level 1), all load is supplied, operating limits are enforced, and no limit violations occur in a contingency. Security level 2 is one where all load is supplied, operating limits are satisfied, and any violations caused by a contingency can be corrected by control action without loss of load. Level 1 security is considered in Dias and El-Hawary (Feb. 1991).

Studies of the effects of load voltage dependence in PF and OPF (Dias and El-Hawary, Sept. 1989) concluded that for PF incorporating load models, the standard solution gives more conservative results with respect to voltages in most cases. However, exceptions have been observed in one test system. Fuel costs much lower than those associated with the standard OPF are obtained by incorporating load models with all voltages free to vary within bounds. This is due to the decrease in the power demand by the reduction of the voltages at buses whose loads are modeled. When quite a few loads are modeled, the minimum fuel costs may be much lower than the corresponding standard OPF fuel cost with a significant decrease in power demand.

A similar effect can be expected when load models are incorporated in security constrained OPF studies. The decrease in the power demand when load models are incorporated in OPF studies may not be desirable under normal operating conditions. This problem can be avoided in a security constrained OPF by incorporating load models during contingencies only. This not only gives results that are more comparable with standard OPF results, but may also give lower fuel costs without lowering the power demand of the intact system. The modeled loads are assumed to be fed by fixed tap transformers and are modeled using an exponential type of load model.

In Dias and El-Harawy (1990), some selected buses were modeled using an exponential type of load model in three cases. In the first, the specified load at modeled buses is obtained with unity voltage. In the second case, the transformer taps have been adjusted to give all industrial-type consumers 1.0 per unit at the low-voltage panel when the high-side voltage corresponds to the standard OPF solution. In the third case, the specified power demand is assumed to take place when the high-side voltages correspond to the intact case of the standard security constrained OPF solution. It is concluded that a decrease in fuel cost can be obtained in some instances when load models are incorporated in security constrained OPF studies during contingencies only. In situations where a decrease in fuel cost is obtained in this manner, the magnitude of decrease depends on the total percentage of load fed by fixed tap transformers and the sensitivity of these loads to modeling. The tap settings of these fixed tap transformers influence the results as well. An increase in fuel cost can also occur in some isolated cases. However, in either case, given accurate load models, optimal power flow solutions that are more accurate than the conventional OPF solutions can be obtained. An alternate approach for normal OPF as well as security constrained OPF is also suggested.

20.4.1 Influence of Fixed Tap Transformer Fed Loads

A standard OPF assumes that all loads are independent of other system variables. This implies that all loads are fed by ULTC transformers that hold the load-side voltage to within a very narrow bandwidth sufficient to justify the assumption of constant loads. However, when some loads are fed by fixed tap transformers, this assumption can result in discrepancies between the standard OPF solution and its observed version. In systems where the average voltage of the system is reasonably above 1.0 per unit (specifically where the loads fed by fixed tap transformers have voltages greater than the voltage at which the specified power demand occurs), the practically observed version of the standard OPF solution will have a higher total power demand, and hence a higher fuel cost, and total revenue, and net revenue. Conversely, where such voltages are lower than the voltage at which the specified power demand occurs, the total power demand, fuel cost, total and net revenues will be lower than expected. For the former case, the system voltages will usually be slightly less than expected, while for the latter case they will usually be slightly higher than expected.

The changes in the power demand at some buses (in the observed version) will alter the power flows on the transmission lines, and this can cause some lines to deliver more power than expected. When this occurs on transmission lines that have power flows near their upper limit, the observed power flows may be above the respective upper limit, causing a security violation. Where the specified power demand occurs at the bus voltages obtained by a standard OPF solution, the observed version of the standard OPF solution will be itself, and there will ideally be no security violations in the observed version.

Most of the above conclusions apply to security constrained OPF as well (Dias and El-Hawary, Nov. 1991). However, since a security constrained OPF solution will in general have higher voltages than its normal counterpart (in order to avoid low voltage limit violations during contingencies), the increase in power demand, and total and net revenues will be more significant while the decrease in the above quantities will be less significant. Also, the security violations due to line flows will now be experienced mainly during contingencies, as most line flows will now usually be below their upper limits for the intact case. For security constrained OPF solutions that incorporate load models only during contingencies, the simulated and observed results will mainly differ in the intact case. Also, with loads modeled during contingencies, the average voltage is lower than for the standard security constrained OPF solution and hence there will be more cases with a decrease in the power demand, fuel cost, and total and net revenues in the observed version of the results than for its standard counterpart.

20.5 Operational Requirements for Online Implementation

The most demanding requirements on OPF technology are imposed by online implementation. It was argued that OPF, as expressed in terms of smooth nonlinear programming formulations, produces results that are far too approximate descriptions of real-life conditions to lead to successful online implementations. Many OPF formulations do not have the capability to incorporate all operational considerations into the solutions. Moreover, some operating practices are occasionally incompatible with such OPF formulations. Consequently, many proposed "theoretical optimal solutions" are of little value to the operators who are almost constantly presented with simultaneous events that are outside the scope of OPF definition. These limitations, if properly addressed, do not have to prevent OPF programs from being used in practice, especially when the operational optimal solution may also not be known. Papalexopoulos (1996) offers some of the requirements that need to be met so that OPF applications are useful to, and usable by, the dispatchers in online applications.

20.5.1 Speed Requirements

Fast OPF programs designed for online application are needed because under normal conditions, the state of the power system changes continuously and can change abruptly during emergency conditions. The changes involve the evolution of bus active and reactive power generation and loads with time, control variables moving to and off their limits as time changes, and topology changes due to switching operations and other planned or forced outages. The need for fast OPF solutions is especially true when an excessive amount of calculations due to modeling of contingency constraints or repeated OPF runs is involved.

In general, an online OPF calculation should have been completed before the state of the power system has changed to another state that is appreciably different from the earlier state. Determining the optimal execution frequency to maximize the benefits of the computations depends on the specific situation and is limited by finite computing resources. It may be preferable to develop incrementally correct and flexible algorithms to offer fast and more frequent scheduling. This leads us to conclude that conventional formulations and algorithms characterized with quadratic convergence that give very accurate and "mathematically optimal" solutions, but neglect operational realities are not appropriate for online implementation. Fast and frequent scheduling requires "hot start" OPF capabilities developed to take advantage of the optimal status of previously optimized operating points. The hot start option is significant when the rate of change of system state is small and previously optimized points are still "relevant" to the current operating conditions.

20.5.2 Robustness of OPF Solutions with Respect to Initial Guess Point

An OPF program needs to produce consistent solutions and thus must not be sensitive to the selected initial guess used. In addition, changes in the OPF solutions between operating states need to be consistent with the changes in the power system operating constraints. The OPF solutions will never be exactly the same when starting from different initial guess points because the solution process is iterative. Any differences should be within the tolerances specified by the convergence criterion, and of a magnitude that would be considered insignificant to the operator. First-order OPF solution methods were not well received because noticeably different solutions could be obtained when an OPF algorithm was initialized from different initial guess points, with only one (or even none) of the solutions actually constituting a local optimum. Theoretically, if the objective function and the feasible region can be shown to be convex, then the optimal solution will be unique (Gill et al., 1981). Unfortunately, the complexity of the nonlinear equations and inequality constraints involved in OPF problems make it untenable to rigorously prove convexity. If multiple local minima actually exist, then additional computational or heuristic methods must be used to resolve the issue.

A normally feasible OPF solution space may become nonconvex (thus leading to multiple OPF solutions) due to two considerations. The first is due to use of discontinuous techniques to model specific operating practices and preferences, and the second is due to modeling of local controls. The conventional power flow problem with local control capability, whose implicit objective is feasible with respect to a limited set of inequalities, does not have a unique solution. Nevertheless, solutions of the same problem from different starting conditions usually match quite closely. Occasionally, different initial guess solutions can lead to different solutions. This takes place when two or more feasible voltage levels can satisfy nonlinear loads. OPF applications, however, should be able to overcome this type of ambiguity.

20.5.3 Discrete Modeling

Discrete control is widely used in the electric network. For example, transformers are used for voltage control, shunt capacitors and reactors are switched on or off to correct voltage profiles and to reduce active power transmission losses, and phase shifters are used to regulate the MW flows of transmission lines. An efficient and effective OPF discretization procedure is needed to assist the operators in utilizing discrete controls in a realistic and optimal or near-optimal manner. Discrete elements to be included in the OPF formulation are branch switching; prohibited zones of generator cost curves; and priority sequence levels for unfeasibility handling. OPF algorithms designed for online applications should be able to appropriately handle the discrete aspects of the problem.

Using both discrete and continuous controls converts the OPF into a mixed discrete-continuous optimization problem. A possible accurate solution using a method such as mixed-integer nonlinear programming would be orders of magnitude slower than ordinary nonlinear programming methods (Gill et al., 1981). Linear programming-based OPF algorithms allow substantial recognition of discrete controls by setting the cost curve segment break points at discrete control steps. However, most methods that solve for a nonseparable objective function by nonlinear programming methods do not properly model discrete controls.

Current OPF algorithms treat all controls as continuous variables during the initial solution process. Once the continuous solution is obtained, each discrete variable is moved to the nearest discrete setting. This produces acceptable solutions, assuming that the step sizes for the discrete controls are sufficiently small, which is usually the case for transformer taps and phase shifter angles (Papalexopoulos et al., 1996). Approximate solutions that can produce near-optimal results appear to be a reasonable alternative to rigorous solution methods. One such scheme (Liu et al., 1991) uses penalty functions for discrete controls. The object is to penalize the continuous approximations of discrete control variables for movements away from their discrete steps. This scheme is well suited for Newton-based OPF algorithms. The scheme consists of a set of rules to determine the timing of introduction and criteria of updating the penalties in the optimization process. This heuristic algorithm is of limited scope. Substantially more work is needed to effectively resolve all problems associated with the discrete nature of controls and other discrete elements of the OPF problem.

20.5.4 Detecting and Handling Infeasibility

As the requirements for satisfactory system operation increase, the region of feasible solutions that satisfy all constraints simultaneously may become too small. In this case, there is a need to establish criteria to prioritize the constraints. For OPF applications, this means that when a feasible solution cannot be found, it is still very important for the algorithm to suggest the "best optimal" engineering solution in some sense, even though it is infeasible. This is even more critical for OPF applications that incorporate contingency constraints.

There are several approaches to deal with this problem. In one approach, all power flow equations are satisfied and only the soft constraints that truly cause the bottlenecks are allowed to be violated using a least squares approximation process. An LP approach introduces a weighted slack variable for each binding constraint. If a constraint can be enforced, the slack variable will be reduced to zero and the constraint will be satisfied. The constraints causing infeasibility will have non-zero slack variables whose magnitudes are proportional to the amounts they need to be relaxed to achieve feasibility. Usually, all binding constraints of a particular type are modeled as if they have identical infeasibility characteristics. That is, all slack variables corresponding to these binding constraints share the same cost curve, and their sensitivities are scaled by a weighting factor associated with the type of the corresponding constraint. Using Newton's method, if the OPF does not converge in the first specified set of iterations, the constraint weighting factors, corresponding to the penalty functions associated with the load bus voltage limits and the branch flow limits, will be reduced successively until a solution is reached. This normally results in all constraints being met except for those load bus voltage and branch flow limits that contribute to infeasibility. Special care should be taken in selecting the proper weighting factors to avoid numerical problems and produce acceptable solutions.

Another approach develops hierarchical rules that operate on the controls and constraints of the OPF problem. The rules introduce discontinuous changes in the original OPF formulation. These changes include using a different set of control/constraint limits, expansion of the control set by class or individually, branch switching, load shedding, etc. They are usually implemented in a predefined priority sequence to be consistent with utility practices. The decision as to when to proceed to the next priority level of modifications to achieve feasibility is critical, especially when it involves radial overloads, normally overloaded constraints and constraints known to have "soft" limits. The selection of a final optimal solution among all the others in the set is achieved with the implementation of a "preference index." An application of the preference index approach that minimizes postcontingency line overloads due to generator outages is given in (Yokoyama et al., 1988).

20.5.5 Consistency of OPF Solutions with Other Online Functions

Online OPF is implemented in either study or closed-loop mode. In study mode, the OPF solutions are presented as recommendations to the operator. In closed-loop mode, control actions are implemented in the system via the SCADA system of the EMS (*IEEE Trans.*, June, 1983). In closed-loop mode, OPF is triggered by a number of events, including an operator request, the execution of the real-time sequence and security analysis, structural change, large load change, etc. A major concern for an OPF in closed-loop mode is the design of its interface with the other online functions, which are executed at different frequencies. Some of these functions are unit commitment, economic dispatch (ED), real-time sequence, security analysis, automatic generation control (AGC), etc. To reduce the discrepancy between ideal and realistic OPF solutions, emphasis should be placed on establishing consistency between these functions and static optimal solutions produced by OPF. This requires proper interfacing and integration of OPF with these functions. The integration design should be flexible enough to allow OPF formulation modifications consistent with the ever dynamic and sometimes ill-defined security problem definition.

20.5.6 Ineffective "Optimal" Rescheduling

Production-grade OPF algorithms use all available control actions to obtain an optimal solution, but for many applications it is not practical to execute more than a limited number of control actions. The OPF problem then becomes one of selecting the best set of actions of a limited size out of a much larger set of possible actions. The problem was identified but no concrete remedies were offered. It is not possible to select the best and most effective set of a given size from existing OPF solutions that use all controls to solve each problem. The control actions cannot be ranked and the effectiveness of an action is not related to its magnitude. Each control facility participates in both minimization of the objective function and enforcement of the constraints. Separation of the two effects for evaluation purposes is not feasible. The problem is difficult to define analytically and existing conventional technologies are not adequate. It is important to note that emerging computational intelligence tools such as fuzzy reasoning and neural networks may offer some resolution. The problem of ineffective rescheduling is related to but is not identical to the "minimum number of controls" objective. It is also closely linked to the problem of discrete control variables, since methods that recognize the discrete nature of some control facilities tend to decrease the number of control actions by keeping inefficient discrete controls at their initial settings.

20.5.7 OPF-Based Transmission Service Pricing

OPF programs are capable of computing marginal costs. Information about the optimal states with respect to changes, such as load variations, operating limit changes, or constraint parameter changes, can be used in many practical applications. Specifically, the sensitivities of the production cost of generation with respect to changes in the bus active power injections are called Bus Incremental Costs (BICs). BICs can be used as nodal prices for pricing transmission services, as they reflect the transmission loss and the congestion components for transferring power from one point to another. In a lossless network with no binding constraints, all BICs should be equal. However, when an operating limit is reached, the congestion component takes effect and all BICs in the network can be different. This means that nodal price differences across uncongested lines can be much larger than marginal losses. Extensive experience has shown that it is possible for power to flow from a bus with higher nodal price to a bus with lower nodal price, resulting in negative transmission charges. Failure to properly account for this effect can lead to unacceptable incentives for transmission users. The same applies in the case of transmission reinforcements to mitigate congestion. If as a result of the upgrades, the incremental transmission rights (positive or negative) are not accounted for properly, similar distortions are possible.

20.6 Conclusions

A review of recent developments in optimal economic operation of electric power systems with emphasis on the optimal power flow formulation was given. We dealt with conventional formulations of economic dispatch, conventional optimal power flow, and accounting for the dependence of the power demand on voltages in the system. Challenges to OPF formulations and solution methodologies for online application were also outlined.

References

Abadie, J. and Carpentier, J., Generalization of the Wolfe reduced gradient method to the case of nonlinear constraints, *Optimization*, R. Fletcher, Ed., Academic Press, New York, 1969, 37–47.

Alsac, O. and Stott, B., Optimal load flow with steady-state security, *IEEE Trans. Power App. Syst.*, PAS-93, 745–751, May/June 1974.

Benthall, T.P., Automatic load scheduling in a multiarea power system, *Proc. IEE*, 115, 592–596, April 1968.

Burchett, R.C., Happ, H.H., Vierath, D.R., and Wirgau, K.A., Developments in optimal power flow, *IEEE Trans. Power App. and Syst.*, PAS-101, 406–414, February 1982.

Burchett, R.C., Happ, H.H., and Vierath, D.R., Quadratically convergent optimal power flow, *IEEE Trans. Power App. and Syst.*, PAS-103, 3267–3275, November 1984.

Carpentier, J., Contribution a l'etude du dispatching economique, *Bull. Soc. Francaise Electriciens*, 8, 431–447, 1962.

Carpentier, J., Differential Injections Method: A general method for secure and optimal load flows, in *IEEE PICA Conference Proceedings* Minneapolis, MN, June 1973, 255–262.

Concordia, C. and Ihara, S., Load representation in power system stability studies, *IEEE Trans. Power App. and Syst.*, PAS-101, 1982.

Contaxis, G.C., Delkis, C., and Korres, G., Decoupled optimal power flow using linear or quadratic programming, *IEEE Trans. Power Syst.*, PWRS-1, 1–7, May 1986.

Dias, L.G. and El-Hawary, M.E., A comparison of load models and their effects on the convergence of Newton power flows, *Inter. J. of Electr. Power and Energy Syst.*, 12, 3–8, 1990.

Dias, L.G. and El-Hawary, M.E., Effects of active and reactive modelling in optimal load flow studies, *IEE Proceedings*, 136, Part C, 259–263, September 1989.

Dias, L.G. and El-Hawary, M.E., Effects of load modeling in security constrained OPF studies, *IEEE Trans. on Power Systems*, 6, 1, 87–93, February 1991.

Dias, L.G. and El-Hawary, M.E., Security constrained OPF: influence of fixed tap transformer loads, *IEEE Trans. on Power Systems*, 6, 1366–1372, November 1991.

Dommel, H.W. and Tinney, W.F., Optimal power flow solutions, *IEEE Trans. Power App. and Syst.*, PAS-87, 1866–1876, October 1968.

El-Din, H.M.Z., Burns, S.D., and Graham, C.E., Voltage/var control with limited control actions, in *Proceedings of the Canadian Electrical Association Spring Meeting*, Toronto, Ontario, March 1989.

El-Hawary, M.E., Power system load modeling and incorporation in load flow solutions, in *Proceedings of the Third Large Systems Symposium*, University of Calgary, June 1982.

El-Hawary, M.E. and Dias, L.G., Bus sensitivity to model parameters in load-flow studies, *IEE Proceedings*, 134, Part C, 302–305 July 1987.

El-Hawary, M.E. and Dias, L.G., Incorporation of load models in load flow studies: form of model effects, *IEE Proceedings*, 134, Part C, 27–30 January 1987.

El-Hawary, M.E. and Dias, L.G., Selection of buses for detailed modeling in load flow studies, *Electric Machines and Power Syst.*, 12, 83–92, 1987.

El-Kady, M.A., Bell, B.D., Carvalho, V.F., Burchett, R.C., Happ, H.H., and Vierath, D.R., Assessment of real-time optimal voltage control, *IEEE Trans. Power Systems*, PWRS-1, 98–107, May 1986.

Fiacco, A.V. and McCormick, G.P., Computational algorithm for the sequential unconstrained minimization technique for nonlinear programming, *Management Sci.*, 10, 601–617, 1964.

Fletcher, R. and Powell, M.J.D., A rapidly convergent descent method for minimization, *Computer Journal*, 6, 163–168, 1963.

Gill, P.E., Murray, W., and Wright, M.H., *Practical Optimization*, Academic Press, New York, 1981.

Glavitsch, H. and Spoerry, M., Quadratic loss formula for reactive dispatch, *IEEE Trans. Power App. and Syst.*, PAS-102, 3850–3858, December 1983.

Happ, H.H. and Vierath, D.R., The OPF for operations planning and for use on line, in *Proceedings of the Second International Conference on Power Systems Monitoring and Control*, July 1986, 290–295.

IEEE Committee Report, System load dynamic simulation, effects and determination of load constants, *IEEE Trans. Power App. and Syst.*, PAS-92, 600–609, 1973.

IEEE Current Operating Problems Working Group Report, On-line load flows from a system operator's viewpoint, *IEEE Trans. on Power Appar. Syst.*, PWRS-102, 1818–1822, June 1983.

IEEE Working Group Report, The effect of frequency and voltage on power system loads, Paper 31 CP 66-64, IEEE Winter Meeting, New York, 1966.

Iliceto, F., Ceyhan, A., and Ruckstuhl, G., Behavior of loads during voltage dips encountered in stability studies, *IEEE Trans. Power App. and Syst.*, PAS-91, 2470–2479, 1972.

Karmarkar, N., New polynomial-time algorithm for linear programming, *Combinatorica*, 4, 1984, 00.373–395.

Kuhn, H.W. and Tucker, A.W., Nonlinear programming, in *Proc. of Second Berkeley Symposium on Mathematical Statistics and Probability*, University of California, Berkeley, 1951, 481–492.

Lindqvist, A., Bubenko, J.A., and Sjelvgren, D., A generalized reduced gradient methodology for optimal reactive power flows, in *Proceedings of the 8th PSCC*, Helsinki, Finland, 1984.

Liu, E., Papalexopoulos, A.D., and Tinney, W.F., Discrete shunt controls in a Newton optimal power flow, *IEEE Winter Power Meeting 1991*, Paper 91WM 041-4 PWRS.

Lootsma, F.A., Logarithmic programming: a method of solving nonlinear programming problems, *Philips Res. Repts.*, 22, 329–344, 1967.

Maria, G.A. and Findlay, J.A., A Newton optimal power flow program for Ontario hydro EMS, *IEEE Trans. Power Syst.*, PWRS-2, 576–584, Aug. 1987.

Merlin, A., On optimal generation planning in large transmission systems (The Maya Problem), in *Proc. of 4th PSCC*, Grenoble, paper 2.1/6, September 1972.

Momoh, J.A., Application of Quadratic Interior Point Algorithm to Optimal Power Flow, EPRI Final report RP 2473-36 II, March 1992.

Papalexopoulos, A.D., Challenges to an on-line OPF implementation, in *IEEE Tutorial Course Optimal Power Flow: Solution Techniques, Requirements, and Challenges*, Publication 96 TP 111-0, IEEE Power Engineering Society, 1996.

Papalexopoulos, A.D., Imparato, C.F., and Wu, F.F., Large-scale optimal power flow: effects of initialization, decoupling and discretization, *IEEE Trans. on Power Appar. Syst.*, PWRS-4, 748–759, May 1989.

Peschon, J., Bree, D.W., and Hajdu, L.P., Optimal solutions involving system security, in *Proc. of the 7th PICA Conf.*, Boston, MA, 1971, 210–218.

Polak, E. and Ribiere, G., Note sur la convergence de methods de directions conjugees, *Rev. Fr. Inform. Rech. Operation*, 16-R1, 35–43, 1969.

Sasson, A.M., Combined use of the Powell and Fletcher-Powell nonlinear programming methods for optimal load flows, *IEEE Trans. Power App. and Syst.*, PAS-88, 1530–1537, October 1969.

Sasson, A.M., Viloria, F., and Aboytes, F., Optimal load flow solution using the Hessian matrix, *IEEE Trans. Power App. and Syst.*, PAS-92, 31–41, Jan/Feb 1973.

Schnyder, G. and Glavitsch, H., Integrated security control using an optimal power flow and switching concepts, in *IEEE PICA Conf. Proc.*, Montreal, Canada, May 18–22, 1987, 429–436.

Shen, C.M. and Laughton, M.A., Power system load scheduling with security constraints using dual linear programming, *IEE Proceedings*, 117, 2117–2127, Nov. 1970.

Stott, B., Alsac, O., and Monticelli, A.J., Security analysis and optimization, *Proc. IEEE*, 75, 1623–1644, Dec. 1987.

Stott, B. and Hobson, E., Power system security control calculations using linear programming. Parts 1 and 2, *IEEE Trans. Power App. and Syst.*, PAS-97, 1713–1731, Sept./Oct. 1978.

Sun, D.I., Ashley, B., Brewer, B., Hughes, A., and Tinney, W.F., Optimal power flow by Newton approach, *IEEE Trans. Power App. and Syst.*, PAS-103, 2864–2880, Oct. 1984.

Vaahedi, E. and El-Din, H.M.Z., Considerations in applying optimal power flow to power systems operation, *IEEE Trans. Power Syst.*, PWRS-4, 694–703, May 1989.

Vaahedi, E., El-Kady, M., Libaque-Esaine, J.A., and Carvalho, V.F., Load models for large scale stability studies from end user consumption, *IEEE Trans. Power Syst.*, PWRS-2, 864–871, 1987.

Vargas, L.S., Quintana, V.H., and Vannelli, A., A tutorial description of an interior point method and its applications to security-constrained economic dispatch, *IEEE Trans. on Power Systems*, 8, 1315–1323, 1993.

Wells, D.W., Method for economic secure loading of a power system, *IEE Proceedings*, 115, 1190–1194, Aug. 1968.

Wolfe, P., Methods of nonlinear programming, *Nonlinear Programming*, J. Abadie, Ed., North Holland, Amsterdam, 1967, 97–131.

Yan, X. and Quintana, V.H., Improving an interior-point-based OPF by dynamic adjustments of step sizes and tolerances, *IEEE Trans. on Power Systems*, 14, 709–717, 1999.

Yokoyama, R., Bae, S.H., Morita, T., and Sasaki, H., Multiobjective optimal generation dispatch based on. probability security criteria, *IEEE Trans. on Power Appar. Syst.*, PWRS-3, 317–324, 1988.

Yu, D.C., Fagan, J.E., Foote, B., and Aly, A.A., An optimal load flow study by the generalized reduced gradient approach, *Electric Power Systems Research*, 10, 47–53, 1986.

Zangwill, W.I., Non-linear programming via penalty functions, *Management Sci.*, 13, 344–358, 1967.