6 Generation with Limited Energy Supply

6.1 INTRODUCTION

The economic operation of a power system requires that expenditures for fuel be minimized over a period of time. When there is no limitation on the fuel supply to any of the plants in the system, the economic dispatch can be carried out with only the present conditions as data in the economic dispatch algorithm. In such a case, the fuel costs are simply the incoming price of fuel with, perhaps, adjustments for fuel handling and maintenance of the plant.

When the energy resource available to a particular plant (be it coal, oil, gas, water, or nuclear fuel) is a limiting factor in the operation of the plant, the entire economic dispatch calculation must be done differently. Each economic dispatch calculation must account for what happened before and what will happen in the future.

This chapter begins the development of solutions to the dispatching problem "over time." The techniques used are an extension of the familiar Lagrange formulation. Concepts involving slack variables and penalty functions are introduced to allow solution under certain conditions.

The example chosen to start with is a fixed fuel supply that must be paid for, whether or not it is consumed. We might have started with a limited fuel supply of natural gas that must be used as boiler fuel because it has been declared as "surplus." The take-or-pay fuel supply contract is probably the simplest of these possibilities.

Alternatively, we might have started directly with the problem of economic scheduling of hydroelectric plants with their stored supply of water or with light-water-moderated nuclear reactors supplying steam to drive turbine generators. Hydroelectric plant scheduling involves the scheduling of water flows, impoundments (storage), and releases into what usually prove to be a rather complicated hydraulic network (namely, the watershed). The treatment of nuclear unit scheduling requires some understanding of the physics involved in the reactor core and is really beyond the scope of this current text (the methods useful for optimizing the unit outputs are, however, quite similar to those used in scheduling other limited energy systems).

6.2 TAKE-OR-PAY FUEL SUPPLY CONTRACT

Assume there are N normally fueled thermal plants plus one turbine generator, fueled under a "*take-or-pay*" *agreement*. We will interpret this type of agreement as being one in which the utility agrees to use a minimum amount of fuel during a period (the "take") or, failing to use this amount, it agrees to pay the minimum charge. This last clause is the "pay" part of the "take-or-pay" contract.

While this unit's cumulative fuel consumption is below the minimum, the system excluding this unit should be scheduled to minimize the total fuel cost, subject to the constraint that the total fuel consumption for the period for this particular unit is equal to the specified amount. Once the specified amount of fuel has been used, the unit should be scheduled normally. Let us consider a special case where the minimum amount of fuel consumption is also the maximum. The system is shown in Figure 6.1. We will consider the operation of the system over $j_{\max}$ time intervals j where $j = 1, \ldots, j_{\max}$, so that

$$P_{1j}, P_{2j}, \ldots, P_{Tj} \quad \text{(power outputs)}$$

$$F_{1j}, F_{2j}, \ldots, F_{Nj} \quad \text{(fuel cost rate)}$$

and

$$q_{T1}, q_{T2}, \ldots, q_{Tj} \quad \text{(take-or-pay fuel input)}$$

are the power outputs, fuel costs, and take-or-pay fuel inputs, where

$P_{ij} \triangleq$ power from i^{th} unit in the j^{th} time interval

$F_{ij} \triangleq$ ₽/h cost for i^{th} unit during the j^{th} time interval

$q_{Tj} \triangleq$ fuel input for unit T in j^{th} time interval

$F_{Tj} \triangleq$ ₽/h cost for unit T in j^{th} time interval

$P_{\text{load } j} \triangleq$ total load in the j^{th} time interval

$n_j \triangleq$ Number of hours in the j^{th} time interval

Mathematically, the problem is as follows:

$$\min \sum_{j=1}^{j_{\max}} \left(n_j \sum_{i=1}^{N} F_{ij} \right) + \sum_{j=1}^{j_{\max}} n_j F_{Tj} \tag{6.1}$$

subject to

$$\phi = \sum_{j=1}^{j_{\max}} n_j q_{Tj} - q_{\text{TOT}} = 0 \tag{6.2}$$

and

$$\psi_j = P_{\text{load } j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0 \qquad \text{for } j = 1 \ldots j_{\max} \tag{6.3}$$

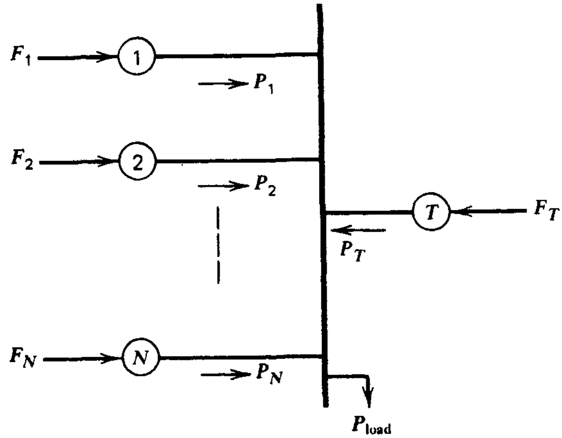


FIG. 6.1 $N + 1$ unit system with take-or-pay fuel supply at unit T.

or, in words,

> We wish to determine the minimum production cost for units 1 to N subject to constraints that ensure that fuel consumption is correct and also subject to the set of constraints to ensure that power supplied is correct each interval.

Note that (for the present) we are ignoring high and low limits on the units themselves. It should also be noted that the term

$$\sum_{j=1}^{j_{max}} n_j F_{Tj}$$

is constant because the total fuel to be used in the "T" plant is fixed. Therefore, the total cost of that fuel will be constant and we can drop this term from the objective function.

The Lagrange function is

$$\mathscr{L} = \sum_{j=1}^{j_{max}} n_j \sum_{i=1}^{N} F_{ij} + \sum_{j=1}^{j_{max}} \lambda_j \left(P_{\text{load}\, j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) + \gamma \left(\sum_{j=1}^{j_{max}} n_j q_{Tj} - q_{\text{TOT}} \right) \quad (6.4)$$

The independent variables are the powers P_{ij} and P_{Tj}, since $F_{ij} = F_i(P_{ij})$ and

$q_{Tj} = q_T(P_{Tj})$. For any given time period, $j = k$,

$$\frac{\partial \mathscr{L}}{\partial P_{ik}} = 0 = n_k \frac{\mathrm{d}F_{ik}}{\mathrm{d}P_{ik}} - \lambda_k \qquad \text{for } i = 1 \ldots N \tag{6.5}$$

and

$$\frac{\partial \mathscr{L}}{\partial P_{Tk}} = -\lambda_k + \gamma n_k \frac{\mathrm{d}q_{Tk}}{\mathrm{d}P_{Tk}} = 0 \tag{6.6}$$

Note that if one analyzes the dimensions of γ, it would be ₱ per unit of q (e.g., ₱/ft^3, ₱/bbl, ₱/ton). As such, γ has the units of a "fuel price" expressed in volume units rather than MBtu as we have used up to now. Because of this, γ is often referred to as a "pseudo-price" or "shadow price." In fact, once it is realized what is happening in this analysis, it becomes obvious that we could solve fuel-limited dispatch problems by simply adjusting the price of the limited fuel(s); thus, the terms "pseudo-price" and "shadow price" are quite meaningful.

Since γ appears unsubscripted in Eq. 6.6, γ would be expected to be a constant value over all the time periods. This is true unless the fuel-limited machine is constrained by fuel-storage limitations. We will encounter such limitations in hydroplant scheduling in Chapter 7. The appendix to Chapter 7 shows when to expect a constant γ and when to expect a discontinuity in γ.

Figure 6.2a shows how the load pattern may look. The solution to a fuel-limited dispatching problem will require dividing the load pattern into time intervals, as in Figure 6.2b, and assuming load to be constant during each interval. Assuming all units are on-line for the period, the optimum dispatch could be done using a simple search procedure for γ, as is shown in Figure 6.3. Note that the procedure shown in Figure 6.3 will only work if the fuel-limited unit does not hit either its high or its low limit in any time interval.

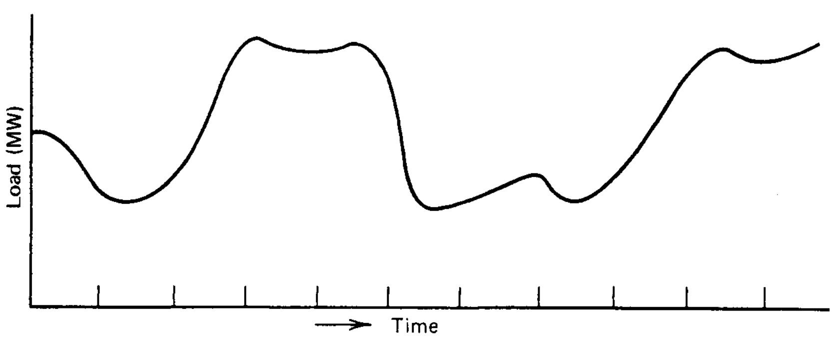


FIG. 6.2a Load pattern.

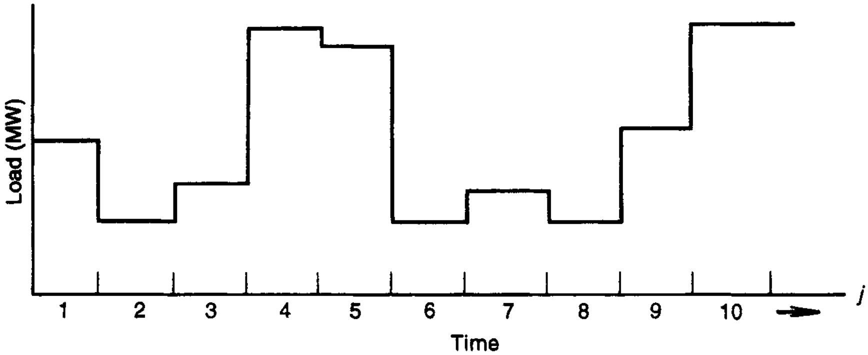


FIG. 6.2b Discrete load pattern.

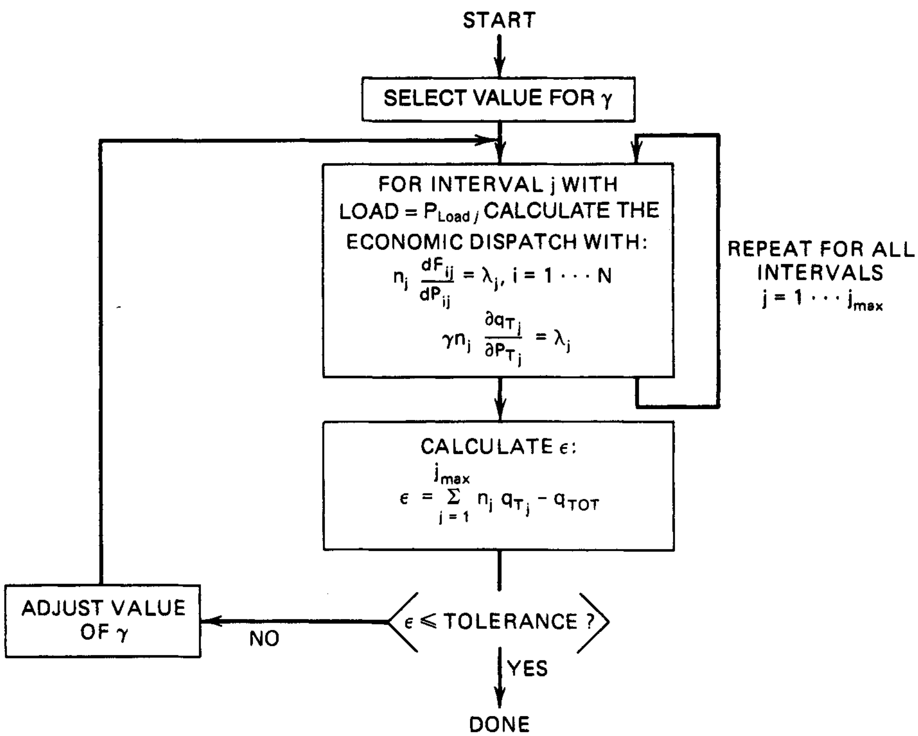


FIG. 6.3 Gamma search method.

6.3 COMPOSITE GENERATION PRODUCTION COST FUNCTION

A useful technique to facilitate the take-or-pay fuel supply contract procedure is to develop a composite generation production cost curve for all the non-fuel-constrained units. For example, suppose there were N non-fuel constrained units to be scheduled with the fuel-constrained unit as shown in Figure 6.4. Then a composite cost curve for units 1, 2, ..., N can be developed.

$$F_s(P_s) = F_1(P_1) + \ldots + F_N(P_N) \tag{6.7}$$

where

$$P_s = P_1 + \ldots + P_N$$

and

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \ldots = \frac{dF_N}{dP_N} = \lambda$$

If one of the units hits a limit, its output is held constant, as in Chapter 3, Eq. 3.6.

A simple procedure to allow one to generate $F_s(P_s)$ consists of adjusting λ from λ^{min} to λ^{max} in specified increments, where

$$\lambda^{min} = \min\left(\frac{dF_i}{dP_i}, i = 1 \ldots N\right)$$

$$\lambda^{max} = \max\left(\frac{dF_i}{dP_i}, i = 1 \ldots N\right)$$

At each increment, calculate the total fuel consumption and the total power output for all the units. These points represent points on the $F_s(P_s)$ curve. The points may be used directly by assuming $F_s(P_s)$ consists of straight-line segments between the points, or a smooth curve may be fit to the points using a least-squares fitting program. Be aware, however, that such smooth curves may have undesirable properties such as nonconvexity (e.g., the first derivative is not monotonically increasing). The procedure to generate the points on $F_s(P_s)$ is shown in Figure 6.5.

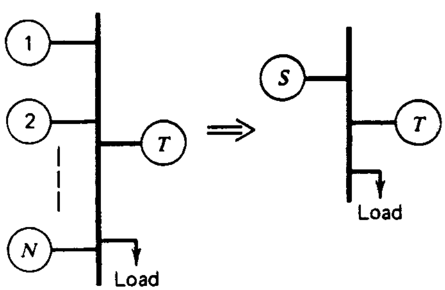


FIG. 6.4 Composite generator unit.

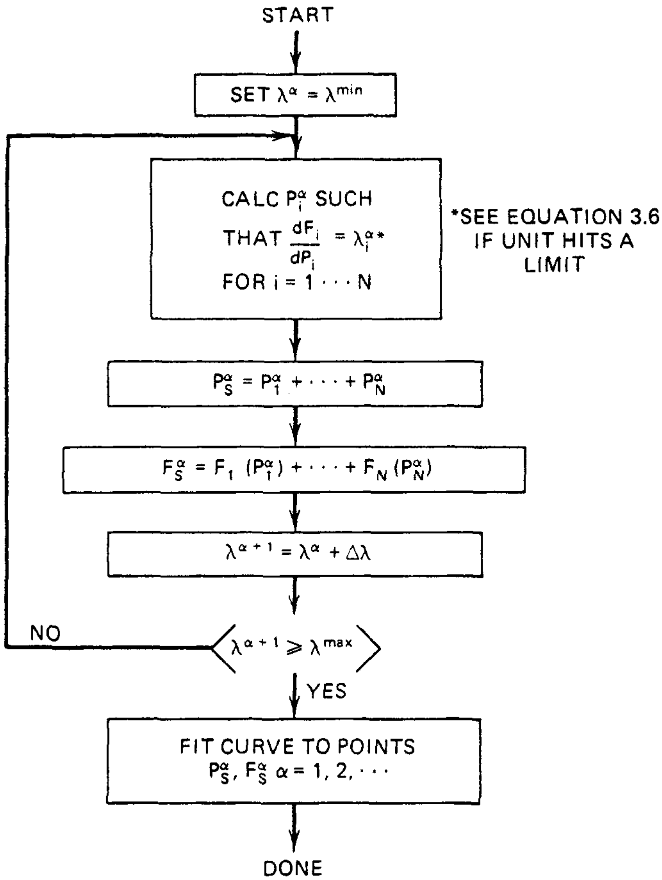


FIG. 6.5 Procedure for obtaining composite cost curve.

EXAMPLE 6A

The three generating units from Example 3A are to be combined into a composite generating unit. The fuel costs assigned to these units will be

Fuel cost for unit 1 = 1.1 ℞/MBtu

Fuel cost for unit 2 = 1.4 ℞/MBtu

Fuel cost for unit 3 = 1.5 ℞/MBtu

Figure 6.6a shows the individual unit incremental costs, which range from 8.3886 to 14.847 ℞/MWh. A program was written based on Figure 6.5, and λ was stepped from 8.3886 to 14.847.

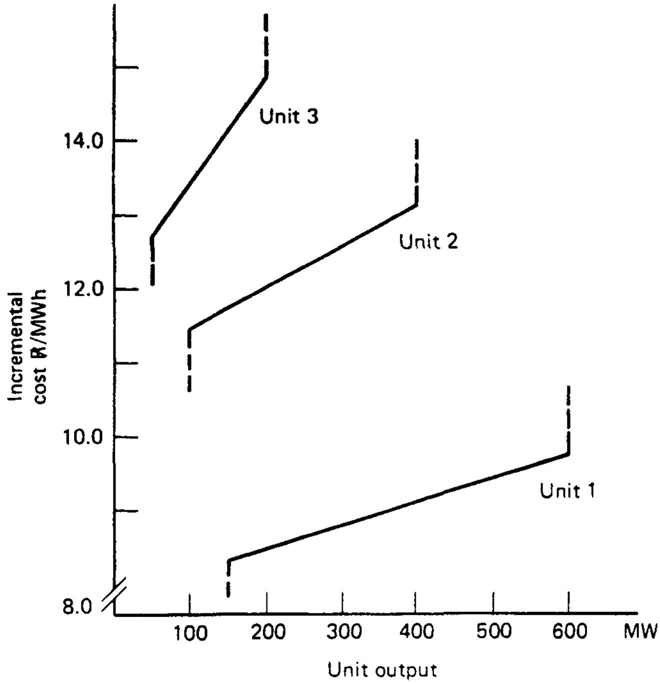


FIG. 6.6a Unit incremental costs.

TABLE 6.1 Lambda Steps Used in Constructing a Composite Cost Curve for Example 6A

Step	λ	P_s	F_s	F_s Approx
1	8.3886	300.0	4077.12	4137.69
2	8.7115	403.4	4960.92	4924.39
3	9.0344	506.7	5878.10	5799.07
4	9.3574	610.1	6828.66	6761.72
5	9.6803	713.5	7812.59	7812.35
6	10.0032	750.0	8168.30	8204.68
7	11.6178	765.6	8348.58	8375.29
8	11.9407	825.0	9048.83	9044.86
9	12.2636	884.5	9768.28	9743.54
10	12.5866	943.9	10506.92	10471.31
11	12.9095	1019.4	11469.56	11436.96
12	13.2324	1088.4	12369.40	12360.58
13	13.5553	1110.67	12668.51	12668.05
14	13.8782	1133.00	12974.84	12979.63
15	14.2012	1155.34	13288.37	13295.30
16	14.5241	1177.67	13609.12	13615.09
17	14.8470	1200.00	13937.00	13938.98

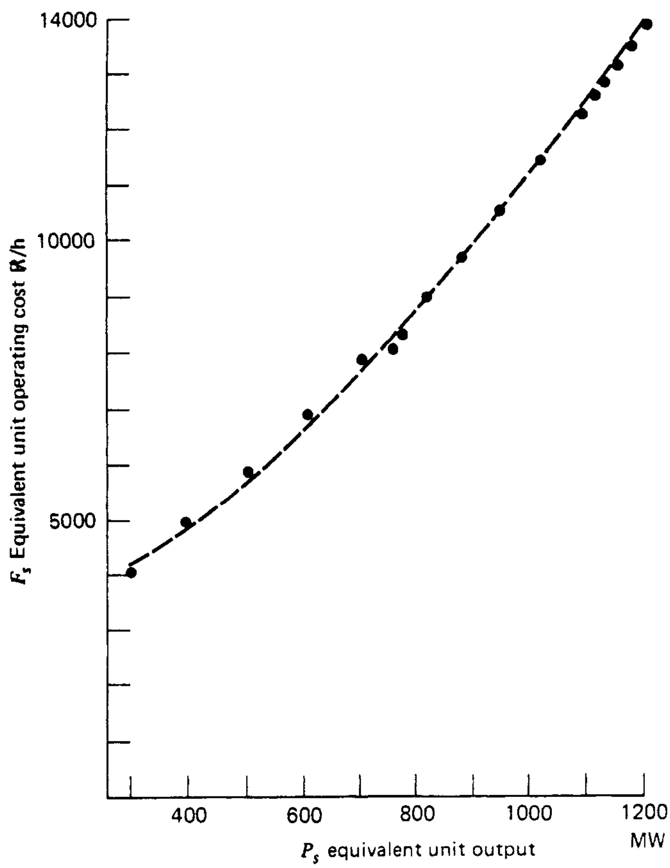


FIG. 6.6b Equivalent unit input/output curve.

At each increment, the three units are dispatched to the same λ and then outputs and generating costs are added as shown in Figure 6.5. The results are given in Table 6.1. The result, called F_s approx in Table 6.1 and shown in Figure 6.6b, was calculated by fitting a second-order polynomial to the P_s and F_s points using a least-squares fitting program. The equivalent unit function is

$$F_s \text{ approx}(P_s) = 2352.65 + 4.7151 P_s + 0.0041168 P_s^2$$
$$(\text{℞/h}) \qquad 300 \text{ MW} \leq P_s \leq 1200 \text{ MW}$$

The reader should be aware that when fitting a polynomial to a set of points, many choices can be made. The preceding function is a good fit to the total operating cost of the three units, but it is not that good at approximating the incremental cost. More-advanced fitting methods should be used if one desires to match total operating cost as well as incremental cost. See Problem 6.2 for an alternative procedure.

EXAMPLE 6B

Find the optimal dispatch for a gas-fired steam plant given the following.

Gas-fired plant:

$$H_T(P_T) = 300 + 6.0P_T + 0.0025P_T^2 \text{ MBtu/h}$$

Fuel cost for gas = 2.0 Ɽ/ccf (where 1 ccf = 10^3 ft^3)

The gas is rated at 1100 Btu/ft^3

$$50 \leq P_T \leq 400$$

Composite of remaining units:

$$H_s(P_s) = 200 + 8.5P_s + 0.002P_s^2 \text{ MBtu/h}$$

Equivalent fuel cost = 0.6 Ɽ/MBtu

$$50 \leq P_s \leq 500$$

The gas-fired plant must burn $40 \cdot 10^6$ ft^3 of gas. The load pattern is shown in Table 6.2. If the gas constraints are ignored, the optimum economic schedule for these two plants appears as is shown in Table 6.3. Operating cost of the composite unit over the entire 24-h period is 52,128.03 Ɽ. The total gas consumption is $21.8 \cdot 10^6$ ft^3. Since the gas-fired plant must burn $40 \cdot 10^6$ ft^3 of gas, the cost will be 2.0 Ɽ/1000 ft^3 × $40 \cdot 10^6$ ft^3, which is 80,000 Ɽ for the gas. Therefore, the total cost will be 132,128.03 Ɽ. The solution method shown in Figure 6.3 was used with γ values ranging from 0.500 to 0.875. The final value for γ is 0.8742 Ɽ/ccf with an optimal schedule as shown in Table 6.4. This schedule has a fuel cost for the composite unit of 34,937.47 Ɽ. Note that the gas unit is run much harder and that it does not hit either limit in the optimal

TABLE 6.2 Load Pattern

Time Period	Load
1. 0000–0400	400 MW
2. 0400–0800	650 MW
3. 0800–1200	800 MW
4. 1200–1600	500 MW
5. 1600–2000	200 MW
6. 2000–2400	300 MW

Where: $n_j = 4$, $j = 1 \ldots 6$.

TABLE 6.3 Optimum Economic Schedule (Gas Constraints Ignored)

Time Period	P_s	P_T
1	350	50
2	500	150
3	500	300
4	450	50
5	150	50
6	250	50

TABLE 6.4 Optimal Schedule (Gas Constraints Met)

Time Period	P_s	P_T
1	197.3	202.6
2	353.2	296.8
3	446.7	353.3
4	259.7	240.3
5	72.6	127.4
6	135.0	165.0

schedule. Further, note that the total cost is now

$$34{,}937.47\ \text{R} + 80{,}000\ \text{R} = 114{,}937.4\ \text{R}$$

so we have lowered the total fuel expense by properly scheduling the gas plant.

6.4 SOLUTION BY GRADIENT SEARCH TECHNIQUES

An alternative solution procedure to the one shown in Figure 6.3 makes use of Eqs. 6.5 and 6.6.

$$n_k \frac{dF_{ik}}{dP_{ik}} = \lambda_k$$

and

$$\lambda_k = \gamma n_k \frac{dq_{Tk}}{dP_{Tk}}$$

then

$$\gamma = \left(\frac{\dfrac{dF_{ik}}{dP_{ik}}}{\dfrac{dq_{Tk}}{dP_{Tk}}} \right) \tag{6.8}$$

For an optimum dispatch, γ will be constant for all hours j, $j = 1 \ldots j_{\max}$.

We can make use of this fact to obtain an optimal schedule using the procedures shown in Figure 6.7a or Figure 6.7b. Both these procedures attempt to adjust fuel-limited generation so that γ will be constant over time. The algorithm shown in Figure 6.7a differs from the algorithm shown in Figure 6.7b in the way the problem is started and in the way various time intervals are

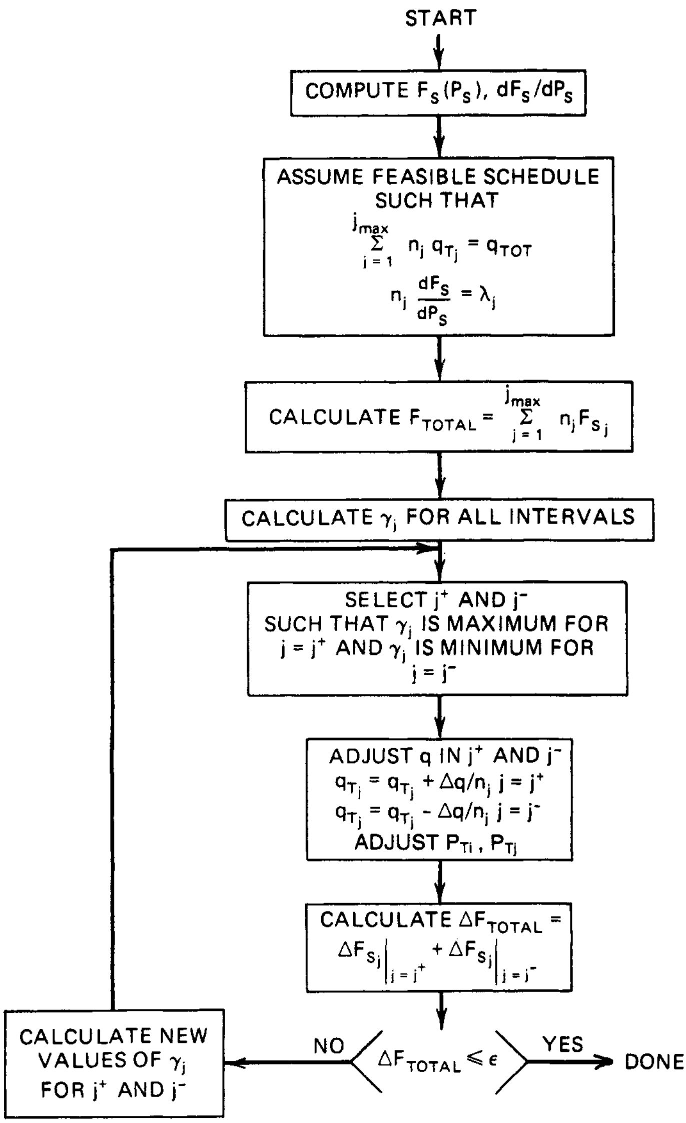


FIG. 6.7a Gradient method based on relaxation technique.

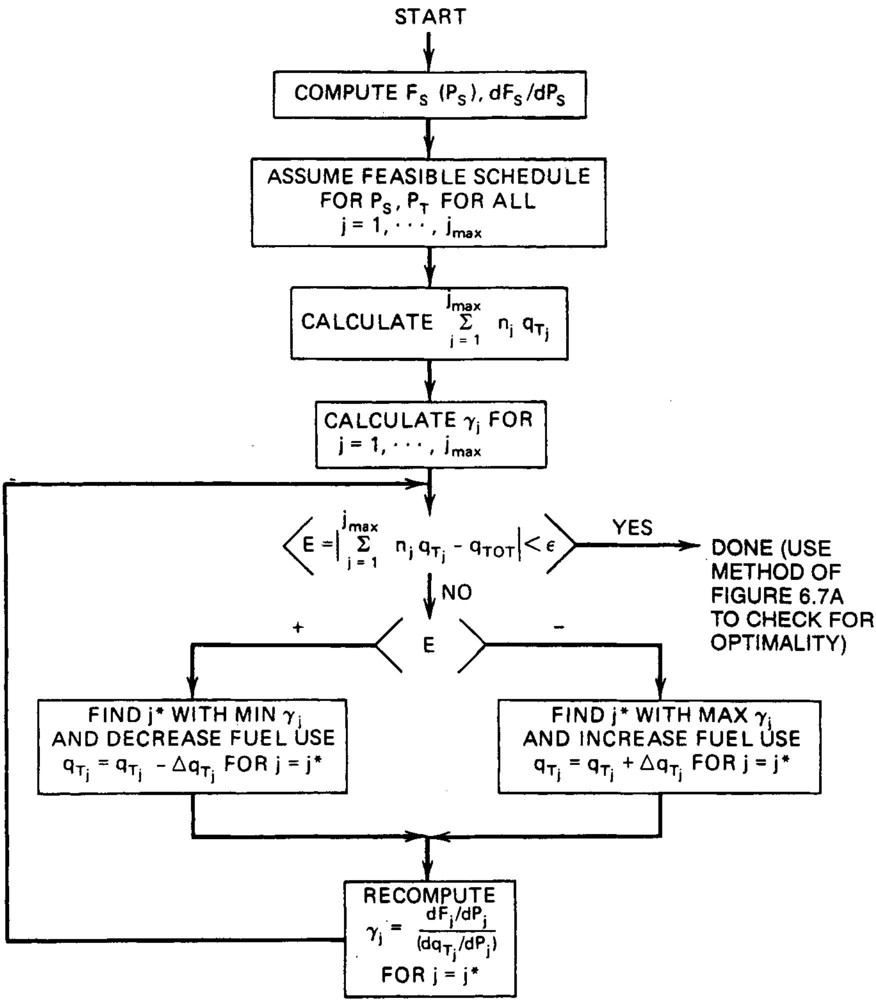


FIG. 6.7b Gradient method based on a simple search.

selected for adjustment. The algorithm in Figure 6.7a requires an initial feasible but not optimal schedule and then finds an optimal schedule by "pairwise" trade-offs of fuel consumption while maintaining problem feasibility. The algorithm in Figure 6.7b does not require an initial feasible fuel usage schedule but achieves this while optimizing. These two methods may be called gradient methods because q_{Tj} is treated as a vector and the γ_j values indicate the gradient of the objective function with respect to q_{Tj}. The method of Figure 6.7b should be followed by that of Figure 6.7a to insure optimality.

EXAMPLE 6C

Use the method of Figure 6.7b to obtain an optimal schedule for the problem given in Example 6B. Assume that the starting schedule is the economic dispatch schedule shown in Example 6B.

Initial Dispatch

	Time Period					
	1	2	3	4	5	6
P_s	350	500	500	450	150	250
P_T	50	150	300	50	50	50
γ	1.0454	1.0266	0.9240	1.0876	0.9610	1.0032

$\sum q_T = 21.84 \cdot 10^6 \text{ ft}^3$.

Since we wish to burn $40.0 \cdot 10^6 \text{ ft}^3$ of gas, the error is negative; therefore, we must increase fuel usage in the time period having maximum γ, that is, period 4. As a start, increase P_T to 150 MW and drop P_s to 350 MW in period 4.

Result of Step 1

	Time Period					
	1	2	3	4	5	6
P_s	350	500	500	350	150	250
P_T	50	150	300	150	50	50
γ	1.0454	1.0266	0.9240	0.9680	0.9610	1.0032

$\sum q_T = 24.2 \cdot 10^6 \text{ ft}^3$.

The error is still negative, so we must increase fuel usage in the period with maximum γ, which is now period 1. Increase P_T to 200 MW and drop P_s to 200 MW in period 1.

Result of Step 2

	Time Period					
	1	2	3	4	5	6
P_s	200	500	500	350	150	250
P_T	200	150	300	150	50	50
γ	0.8769	1.0266	0.9240	0.9680	0.9610	1.0032

$\sum q_T = 27.8 \cdot 10^6 \text{ ft}^3$.

and so on. After 11 steps, the schedule looks like this:

	Time Period					
	1	2	3	4	5	6
P_s	200	350	450	250	75	140
P_T	200	300	350	250	125	160
γ	0.8769	0.8712	0.8772	0.8648	0.8767	0.8794

$\sum q_T = 40.002 \cdot 10^6\ \text{ft}^3$.

which is beginning to look similar to the optimal schedule generated in Example 6A.

6.5 HARD LIMITS AND SLACK VARIABLES

This section takes account of hard limits on the take-or-pay generating unit. The limits are

$$P_T \geq P_{T\,\mathrm{min}} \tag{6.9}$$

and

$$P_T \leq P_{T\,\mathrm{max}} \tag{6.10}$$

These may be added to the Lagrangian by the use of two constraint functions and two new variables called *slack variables* (see Appendix 3A). The constraint functions are

$$\psi_{1j} = P_{Tj} - P_{T\,\mathrm{max}} + S_{1j}^2 \tag{6.11}$$

and

$$\psi_{2j} = P_{T\,\mathrm{min}} - P_{Tj} + S_{2j}^2 \tag{6.12}$$

where S_{1j} and S_{2j} are slack variables that may take on any real value including zero.

The new Lagrangian then becomes

$$\mathscr{L} = \sum_{j=1}^{j_{\mathrm{max}}} n_j \sum_{i=1}^{N} F_{ij} + \sum_{j=1}^{j_{\mathrm{max}}} \lambda_j \left(P_{\mathrm{load}\,j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) + \gamma \left(\sum_{j=1}^{j_{\mathrm{max}}} n_j q_{Tj} - Q_{\mathrm{TOT}} \right)$$
$$+ \sum_{j=1}^{j_{\mathrm{max}}} \alpha_{1j}(P_{Tj} - P_{T\,\mathrm{max}} + S_{1j}^2) + \sum_{j=1}^{j_{\mathrm{max}}} \alpha_{2j}(P_{T\,\mathrm{min}} - P_{Tj} + S_{2j}^2) \tag{6.13}$$

where α_{1j}, α_{2j} are Lagrange multipliers. Now, the first partial derivatives for the k^{th} period are

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial P_{ik}} &= 0 = n_k \frac{\mathrm{d}F_{ik}}{\mathrm{d}P_{ik}} - \lambda_k \\
\frac{\partial \mathscr{L}}{\partial P_{Tk}} &= 0 = -\lambda_k + \alpha_{1k} - \alpha_{2k} + \gamma n_k \frac{\mathrm{d}q_{T_k}}{\mathrm{d}P_{T_k}} \\
\frac{\partial \mathscr{L}}{\partial S_{1k}} &= 0 = 2\alpha_{1k} S_{1k} \\
\frac{\partial \mathscr{L}}{\partial S_{2k}} &= 0 = 2\alpha_{2k} S_{2k}
\end{aligned} \tag{6.14}$$

As we noted in Appendix 3A, when the constrained variable (P_{T_k} in this case) is within bounds, the new Lagrange multipliers $\alpha_{1k} = \alpha_{2k} = 0$ and S_{1k} and S_{2k} are nonzero. When the variable is limited, one of the slack variables, S_{1k} or S_{2k}, becomes zero and the associated Lagrange multiplier will take on a nonzero value.

Suppose in some interval k, $P_{Tk} = P_{\max}$, then $S_{1k} = 0$ and $\alpha_{1k} \neq 0$. Thus,

$$-\lambda_k + \alpha_{1k} + \gamma n_k \frac{\mathrm{d}q_{Tk}}{\mathrm{d}P_{Tk}} = 0 \tag{6.15}$$

and if

$$\lambda_k > \gamma n_k \frac{\mathrm{d}q_{Tk}}{\mathrm{d}P_{Tk}}$$

the value of α_{1k} will take on the value just sufficient to make the equality true.

EXAMPLE 6D

Repeat Example 6B with the maximum generation on P_T reduced to 300 MW. Note that the optimum schedule in Example 6A gave a $P_T = 353.3$ MW in the third time period. When the limit is reduced to 300 MW, the gas-fired unit will have to burn more fuel in other time periods to meet the $40 \cdot 10^3$ ft^3 gas consumption constraint.

TABLE 6.5 Resulting Optimal Schedule with $P_{T\max} = 300$ MW

Time Period j	P_{sj}	P_{Tj}	λ_j	$\gamma_{nj} \dfrac{\partial q_T}{\partial P_{Tj}}$	α_{1j}
1	183.4	216.6	5.54	5.54	0
2	350.0	300.0	5.94	5.86	0.08
3	500.0	300.0	6.3	5.86	0.44
4	245.4	254.6	5.69	5.69	0
5	59.5	140.5	5.24	5.24	0
6	121.4	178.6	5.39	5.39	0

Table 6.5 shows the resulting optimal schedule where $\gamma = 0.8603$ and total cost = 122,984.83 ₨.

6.6 FUEL SCHEDULING BY LINEAR PROGRAMMING

Figure 6.8 shows the major elements in the chain making up the delivery system that starts with raw-fuel suppliers and ends up in delivery of electric power to individual customers. The basic elements of the chain are as follows.

The suppliers: These are the coal, oil, and gas companies with which the utility must negotiate contracts to acquire fuel. The contracts are usually written for a long term (10 to 20 yr) and may have stipulations, such as the minimum and maximum limits on the quantity of fuel delivered over a specified time period. The time period may be as long as a year, a month, a week, a day, or even for a period of only a few minutes. Prices may change, subject to the renegotiation provisions of the contracts.

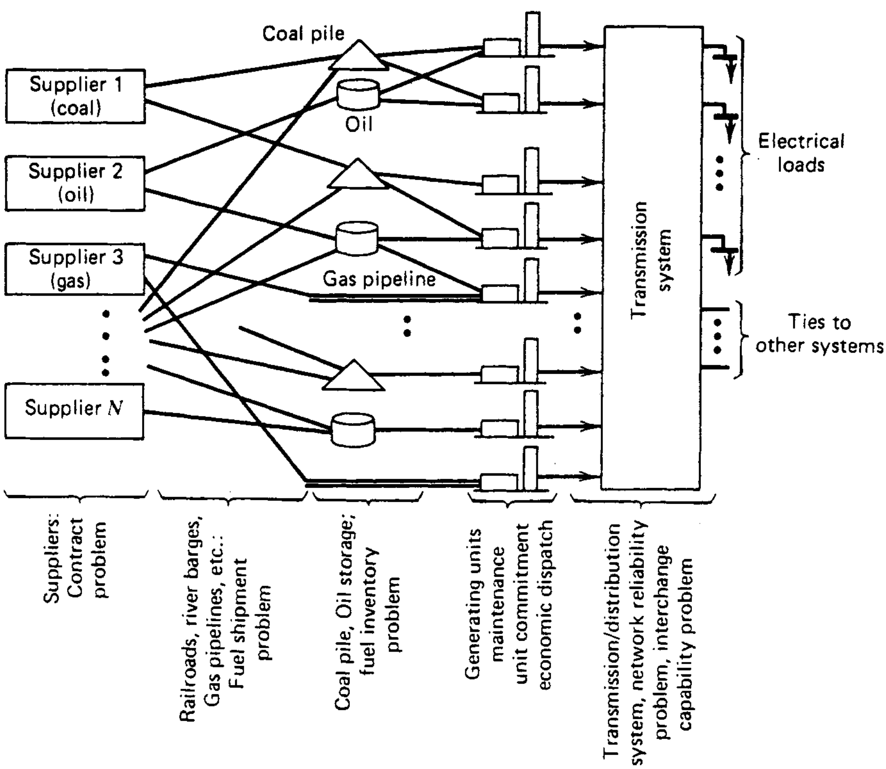


FIG. 6.8 Energy delivery system.

Transportation: Railroads, unit trains, river barges, gas-pipeline companies, and such, all present problems in scheduling of deliveries of fuel.

Inventory: Coal piles, oil storage tanks, underground gas storage facilities. Inventories must be kept at proper levels to forestall fuel shortages when load levels exceed forecast or suppliers or shippers are unable to deliver. Price fluctuations also complicate the decisions on when and how much to add or subtract from inventories.

The remainder of the system—generators, transmission, and loads—are covered in other chapters.

One of the most useful tools for solving large fuel-scheduling problems is linear programming (LP). If the reader is not familiar with LP, an easily understood algorithm is provided in the appendix of this chapter.

Linear programming is an optimization procedure that minimizes a linear objective function with variables that are also subject to linear constraints. Because of this limitation, any nonlinear functions either in the objective or in the constraint equations will have to be approximated by linear or piecewise linear functions.

To solve a fuel-scheduling problem with linear programming, we must break the total time period involved into discrete time increments, as was done in Example 6B. The LP solution will then consist of an objective function that is made up of a sum of linear or piecewise linear functions, each of which is a function of one or more variables from only one time step. The constraints will be linear functions of variables from each time step. Some constraints will be made up of variables drawn from one time step whereas others will span two or more time steps. The best way to illustrate how to set up an LP to solve a fuel-scheduling problem will be to use an example.

EXAMPLE 6E

We are given two coal-burning generating units that must both remain on-line for a 3-wk period. The combined output from the two units is to supply the following loads (loads are assumed constant for 1 wk).

Week	Load (MW)
1	1200
2	1500
3	800

The two units are to be supplied by one coal supplier who is under contract to supply 40,000 tons of coal per week to the two plants. The plants have existing coal inventories at the start of the 3-wk period. We must solve for the following.

1. How should each plant be operated each week?
2. How should the coal deliveries be made up each week?

The data for the problem are as follows.

Coal: Heat value = 11,500 Btu/lb = 23 MBtu/ton (1 ton = 2000 lb)

Coal can all be delivered to one plant or the other or it can be split, some going to one plant, some to the other, as long as the total delivery in each week is equal to 40,000 tons. The coal costs 30 Ɽ/ton or 1.3 Ɽ/MBtu.

Inventories: Plant 1 has an initial inventory of 70,000 tons; its final inventory is not restricted

Plant 2 has an initial inventory of 70,000 tons; its final inventory is not restricted

Both plants have a maximum coal storage capacity of 200,000 tons of coal.

Generating units:

Unit	Min (MW)	Max (MW)	Heat Input at Min (MBtu/h)	Heat Input at Max (MBtu/h)
1	150	600	1620	5340
2	400	1000	3850	8750

The input versus output function will be approximated by a linear function for each unit:

$$H_1(P_1) = 380.0 + 8.267P_1$$

$$H_2(P_2) = 583.3 + 8.167P_2$$

The unit cost curves are

$$F_1(P_1) = 1.3\ \text{Ɽ/MBtu} \times H_1(P_1) = 495.65 + 10.78P_1\ (\text{Ɽ/h})$$

$$F_2(P_2) = 1.3\ \text{Ɽ/MBtu} \times H_2(P_2) = 760.8 + 10.65P_2\ (\text{Ɽ/h})$$

The coal consumption q(tons/h) for each unit is

$$q_1(P_1) = \frac{1}{23}\left(\frac{\text{tons}}{\text{MBtu}}\right) \times H_1(P_1) = 16.52 + 0.3594P_1 \text{ tons/h}$$

$$q_2(P_2) = \frac{1}{23}\left(\frac{\text{tons}}{\text{MBtu}}\right) \times H_2(P_2) = 25.36 + 0.3551P_2 \text{ tons/h}$$

To solve this problem with linear programming, assume that the units are to be operated at a constant rate during each week and that the coal deliveries will each take place at the beginning of each week. Therefore, we will set up the problem with 1-wk time periods and the generating unit cost functions and coal consumption functions will be multiplied by 168 h to put them on a "per week" basis; then,

$$\begin{aligned} F_1(P_1) &= 83{,}269.2 + 1811P_1 \text{ ℞/wk} \\ F_2(P_2) &= 127{,}814.4 + 1789P_2 \text{ ℞/wk} \\ q_1(P_1) &= 2775.4 + 60.4P_1 \text{ tons/wk} \\ q_2(P_2) &= 4260.5 + 59.7P_2 \text{ tons/wk} \end{aligned} \tag{6.16}$$

We are now ready to set up the objective function and the constraints for our linear programming solution.

Objective function: To minimize the operating cost over the 3-wk period. The objective function is

$$\begin{aligned} \text{Minimize } Z = {} & F_1[P_1(1)] + F_2[P_2(1)] + F_1[(P_1(2)] + F_2[P_2(2)] \\ & + F_1[P_1(3)] + F_2[P_2(2)] \end{aligned} \tag{6.17}$$

where $P_i(j)$ is the power output of the i^{th} unit during the j^{th} week, $j = 1 \ldots 3$.

Constraints: During each time period, the total power delivered from the units must equal the scheduled load to be supplied; then

$$\begin{aligned} P_1(1) + P_2(1) &= 1200 \\ P_1(2) + P_2(2) &= 1500 \\ P_1(3) + P_2(3) &= 800 \end{aligned} \tag{6.18}$$

Similarly, the coal deliveries, D_1 and D_2, made to plant 1 and plant 2, respectively, during each week must sum to 40,000 tons; then

$$\begin{aligned} D_1(1) + D_2(1) &= 40{,}000 \\ D_1(2) + D_2(2) &= 40{,}000 \\ D_1(3) + D_2(3) &= 40{,}000 \end{aligned} \tag{6.19}$$

The volume of coal at each plant at the beginning of each week plus the delivery of coal to that plant minus the coal burned at the plant will give the coal remaining at the beginning of the next week. Letting V_1 and V_2 be the volume of coal in each coal pile at the beginning of the week, respectively, we have the following set of equations governing the two coal piles.

$$\begin{aligned} V_1(1) + D_1(1) - q_1(1) &= V_1(2) \\ V_2(1) + D_2(1) - q_2(1) &= V_2(2) \\ V_1(2) + D_1(2) - q_1(2) &= V_1(3) \\ V_2(2) + D_2(2) - q_2(2) &= V_2(3) \\ V_1(3) + D_1(3) - q_1(3) &= V_1(4) \\ V_2(3) + D_2(3) - q_2(3) &= V_2(4) \end{aligned} \tag{6.20}$$

where $V_i(j)$ is the volume of coal in the i^{th} coal pile at the beginning of the j^{th} week.

To set these equations up for the linear-programming solutions, substitute the $q_1(P_1)$ and $q_2(P_2)$ equations from 6.16 into the equations of 6.20. In addition, all constant terms are placed on the right of the equal sign and all variable terms on the left; this leaves the constraints in the standard form for inclusion in the LP. The result is

$$\begin{aligned} D_1(1) - 60.4P_1(1) - V_1(2) &= 2775.4 - V_1(1) \\ D_2(1) - 59.7P_2(1) - V_2(2) &= 4260.5 - V_2(1) \\ V_1(2) + D_1(2) - 60.4P_1(2) - V_1(3) &= 2775.4 \\ V_2(2) + D_2(2) - 59.7P_2(2) - V_2(3) &= 4260.5 \\ V_1(3) + D_1(3) - 60.4P_1(3) - V_1(4) &= 2775.4 \\ V_2(3) + D_2(3) - 59.7P_2(3) - V_2(4) &= 4260.5 \end{aligned} \tag{6.21}$$

Note: $V_1(1)$ and $V_2(1)$ are constants that will be set when we start the problem.

The constraints from Eqs. 6.18, 6.19, and 6.21 are arranged in a matrix, as shown in Figure 6.9. Each variable is given an upper and lower bound in keeping with the "upper bound" solution shown in the appendix of this chapter. The $P_1(t)$ and $P_2(t)$ variables are given the upper and lower bounds corresponding

	Week 1				Week 2						Week 3						Final Conditions		
Problem Variable	D1(1)	P1(1)	D2(1)	P2(1)	V1(2)	D1(2)	P1(2)	V2(2)	D2(2)	P2(2)	V1(3)	D1(3)	P1(3)	V2(3)	D2(3)	P2(3)	V1(4)	V2(4)	Constraint Units
LP Variable	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	
Constraint																			
1		1		1															1200
2	1		1																40000
3	1	−.60.4			−1														2775.4 − $V_1(1)$
4			1	−59.7				−1											4260.5 − $V_2(1)$
5							1			1									1500
6						1			1										40000
7					1	1	−60.4				−1								2775.4
8								1	1	−59.7				−1					4260.5
9													1			1			800
10												1			1				40000
11											1	1	−60.4				−1		2775.4
12														1	1	−59.7		−1	4260.5
Variable min.	0	150	0	400	0	0	150	0	0	400	0	0	150	0	0	400	0	0	
Variable max.	40000	600	40000	1000	200000	40000	600	200000	40000	1000	200000	40000	600	200000	40000	1000	200000	200000	

FIG. 6.9 Linear-programming constraint matrix for Example 6E.

to the upper and lower limits on the generating units. $D_1(t)$ and $D_2(t)$ are given upper and lower bounds of 40,000 and zero. $V_1(t)$ and $V_2(t)$ are given upper and lower bounds of 200,000 and zero.

Solution: The solution to this problem was carried out with a computer program written to solve the upper bound LP problem using the algorithm shown in the Appendix. The first problem solved had coal storage at the beginning of the first week of

$$V_1(1) = 70{,}000 \text{ tons}$$

$$V_2(1) = 70{,}000 \text{ tons}$$

The solution is:

Time Period	V_1	D_1	P_1	V_2	D_2	P_2
1	70000.0	0	200	70000.0	40000.0	1000
2	55144.6	0	500	46039.5	40000.0	1000
3	22169.2	19013.5	150	22079.0	20986.5	650
4	29347.3					

Optimum cost = 6,913,450.8 Ɽ.

In this case, there are no constraints on the coal deliveries to either plant and the system can run in the most economic manner. Since unit 2 has a lower incremental cost, it is run at its maximum when possible. Furthermore, since no restrictions were placed on the coal pile levels at the end of the third week, the coal deliveries could have been shifted a little from unit 2 to unit 1 with no effect on the generation dispatch.

The next case solved was purposely structured to create a fuel shortage at unit 2. The beginning inventory at plant 2 was set to 50,000 tons, and a requirement was imposed that at the end of the third week the coal pile at unit 2 be no less than 8000 tons. The solution was made by changing the right-hand side of the fourth constraint from $-65{,}739.5$ (i.e., $4260.5 - 70{,}000$) to -45739.5 (i.e., $4260.5 - 50{,}000$) and placing a lower bound on $V_2(4)$ (i.e., variable X_{18}) of 8000. The solution is:

Time Period	V_1	D_1	P_1	V_2	D_2	P_2
1	70000.0	0	200	50000.0	40000.0	1000
2	55144.6	0	500	26039.5	40000.0	1000
3	22169.2	0	300.5276	2079.0	40000.0	499.4724
4	1241.9307			8000.0		

Optimum cost = 6,916,762.4 Ɽ.

Note that this solution requires unit 2 to drop off its generation in order to meet the end-point constraint on its coal pile. In this case, all the coal must be delivered to plant 2 to minimize the overall cost.

The final case was constructed to show the interaction of the fuel deliveries and the economic dispatch of the generating units. In this case, the initial coal piles were set to 10,000 tons and 150,000 tons, respectively. Furthermore, a restriction of 30,000 tons minimum in the coal pile at unit 1 at the end of the third week was imposed.

To obtain the most economic operation of the two units over the 3-wk period, the coal deliveries will have to be adjusted to insure both plants have sufficient coal. The solution was obtained by setting the right-hand side of the third and fourth constraint equations to -7224.6 and -145739.5, respectively, as well as imposing a lower bound of 30,000 on $V_1(4)$ (i.e., variable X_{17}). The solution is:

Time Period	V_1	D_1	P_1	V_2	D_2	P_2
1	10000.0	4855.4	200	150000.0	35144.6	1000
2	0.0	40000.0	500	121184.1	0	1000
3	7024.6	40000.0	150	57223.6	0	650
4	35189.2			14158.1		

Optimum cost = 6,913,450.8 ℞.

The LP was able to find a solution that allowed the most economic operation of the units while still directing enough coal to unit 1 to allow it to meet its end-point coal pile constraint. Note that, in practice, we would probably not wish to let the coal pile at unit 1 go to zero. This could be prevented by placing an appropriate lower bound on all the volume variables (i.e., X_5, X_8, X_{11}, X_{14}, X_{17}, and X_{18}).

This example has shown how a fuel-management problem can be solved with linear programming. The important factor in being able to solve very large fuel-scheduling problems is to have a linear-programming code capable of solving large problems having perhaps tens of thousands of constraints and as many, or more, problem variables. Using such codes, elaborate fuel-scheduling problems can be optimized out over several years and play a critical role in utility fuel-management decisions.

APPENDIX
Linear Programming

Linear programming is perhaps the most widely applied mathematical programming technique. Simply stated, linear programming seeks to find the optimum value of a linear objective function while meeting a set of linear constraints. That is, we wish to find the optimum set of x values that minimize the following objective function:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_Nx_N$$

subject to a set of linear constraints:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1N}x_N \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2N}x_N \leq b_2$$
$$\vdots$$

In addition, the variables themselves may have specified upper and lower limits.

$$x_i^{\min} \leq x_i \leq x_i^{\max} \qquad i = 1 \ldots N$$

There are a variety of solutions to the LP problem. Many of these solutions are tailored to a particular type of problem. This appendix will not try to develop the theory of alternate LP solution methods. Rather, it will present a simple LP algorithm that can be used (or programmed on a computer) to solve the applicable power-system sample problems given in this text.

The algorithm is presented in its simplest form. There are alternative formulations, and these will be indicated when appropriate. If the student has access to a standard LP program, such a standard program may be used to solve any of the problems in this book.

The LP technique presented here is properly called an *upper-bounding dual linear programming algorithm*. The "upper-bounding" part of its name refers to the fact that variable limits are handled implicitly in the algorithm. "Dual" refers to the theory behind the way in which the algorithm operates. For a complete explanation of the primal and dual algorithms, refer to the references cited at the end of this chapter.

In order to proceed in an orderly fashion to solve a dual upper-bound linear programming problem, we must first add what is called a *slack variable* to each constraint. The slack variable is so named because it equals the difference or slack between a constraint and its limit. By placing a slack variable into an inequality constraint, we can transform it into an equality constraint. For example, suppose we are given the following constraint.

$$2x_1 + 3x_2 \leq 15 \tag{6A.1}$$

We can transform this constraint to an equality constraint by adding a slack variable, x_3.

$$2x_1 + 3x_2 + x_3 = 15 \tag{6A.2}$$

If x_1 and x_2 were to be given values such that the sum of the first two terms in Eq. 6A.2 added up to less than 15, we could still satisfy Eq. 6A.2 by setting x_3 to the difference. For example, if $x_1 = 1$ and $x_2 = 3$, then $x_3 = 4$ would satisfy Eq. 6A.2. We can go even further, however, and restrict the values of x_3 so that Eq. 6A.2 still acts as an inequality constraint such as Eq. 6A.1. Note that when the first two terms of Eq. 6A.2 add to exactly 15, x_3 must be set to zero. By restricting x_3 to always be a positive number, we can force Eq. 6A.2 to yield the same effect as Eq. 6A.1. Thus,

$$\left.\begin{array}{c} 2x_1 + 3x_2 + x_3 = 15 \\ 0 \le x_3 \le \infty \end{array}\right\} \text{ is equivalent to: } 2x_1 + 3x_2 \le 15$$

For a "greater than or equal to" constraint, we merely change the bounds on the slack variable:

$$\left.\begin{array}{c} 2x_1 + 3x_2 + x_3 = 15 \\ -\infty \le x_3 \le 0 \end{array}\right\} \text{ is equivalent to: } 2x_1 + 3x_2 \ge 15$$

Because of the way the dual upper-bounding algorithm is initialized, we will always require slack variables in every constraint. In the case of an equality constraint, we will add a slack variable and then require its upper and lower bounds to both equal zero.

To solve our linear programming algorithm, we must arrange the objective function and constraints in a tabular form as follows.

$$\begin{array}{llll} a_{11}x_1 + a_{12}x_2 + \ldots + x_{\text{slack}_1} & & & = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots & + x_{\text{slack}_2} & & = b_2 \\ c_1x_1 + c_2x_2 + \ldots & & -Z & = 0 \end{array} \qquad (6A.3)$$

$$\underbrace{\qquad\qquad\qquad}_{\text{Basis variables}}$$

Because we have added slack variables to each constraint, we automatically have arranged the set of equations into what is called *canonical* form. In canonical form, there is at least one variable in each constraint whose coefficient is zero in all the other constraints. These variables are called the *basis variables*. The entire solution procedure for the linear programming algorithm centers on performing "pivot" operations that can exchange a nonbasis variable for a basis variable. A pivot operation may be shown by using our tableau in Eq. 6A.3. Suppose we wished to exchange variable x_1, a nonbasis variable, for x_{slack_2}, a slack variable. This could be accomplished by "pivoting" on column 1, row 2. To carry out the pivoting operation we execute the following steps.

Pivoting on Column 1, Row 2

Step 1 Multiply row 2 by $1/a_{21}$. That is, each a_{2j}, $j = 1 \ldots N$ in row 2 becomes

$$a'_{2j} = \frac{a_{2j}}{a_{21}} \qquad j = 1 \ldots N$$

and

$$b_2 \text{ becomes } b'_2 = \frac{b_2}{a_{21}}$$

Step 2 For each row i ($i \neq 2$), multiply row 2 by a_{i1} and subtract from row i. That is, each coefficient a_{ij} in row i ($i \neq 2$) becomes

$$a'_{ij} = a_{ij} - a_{i1}a'_{2j} \qquad j = 1 \ldots N$$

and

$$b_i \text{ becomes } b'_i = b_i - a_{i1}b'_2$$

Step 3 Last of all, we also perform the same operations in step 2 on the cost row. That is, each coefficient c_j becomes

$$c'_j = c_j - c_1 a'_{2j} \qquad j = 1 \ldots N$$

The result of carrying out the pivot operation will look like this:

$$\begin{aligned}
a'_{12}x_2 + \ldots x_{\text{slack}_1} + a'_{1s_2}x_{\text{slack}_2} &= b'_1 \\
x_1 + a'_{22}x_2 + \ldots \qquad + a'_{2s_2}x_{\text{slack}_2} &= b'_2 \\
c'_2 x_2 + \ldots \qquad + c'_{s2}x_{\text{slack}_2} - Z &= Z'
\end{aligned}$$

Notice that the new basis for our tableau is formed by variable x_1 and x_{slack_1}, x_{slack_2} no longer has zero coefficients in row 1 or the cost row.

The dual upper-bounding algorithm proceeds in simple steps wherein variables that are in the basis are exchanged for variables out of the basis. When an exchange is made, a pivot operation is carried out at the appropriate row and column. The nonbasis variables are held equal to either their upper or their lower value, while the basis variables are allowed to take any value without respect to their upper or lower bounds. The solution terminates when all the basis variables are within their respective limits.

In order to use the dual upper-bound LP algorithm, follow these rules.

Start:

1. Each variable that has a nonzero coefficient in the cost row (i.e., the objective function) must be set according to the following rule.

$$\text{If } C_j > 0, \qquad \text{set } x_j = x_j^{\min}$$
$$\text{If } C_j < 0, \qquad \text{set } x_j = x_j^{\max}$$

2. If $C_j = 0$, x_j may be set to any value, but for convenience set it to its minimum also.
3. Add a slack variable to each constraint. Using the x_j values from steps 1 and 2, set the slack variables to make each constraint equal to its limit.

Variable Exchange:

1. Find the basis variable with the greatest violation; this determines the row to be pivoted on. Call this row R. If there are no limit violations among the basis variables, we are done. The most-violated variable leaves the basis and is set equal to the limit that was violated.
2. Select the variable to enter the basis using one of the following column selection procedures.

Column Selection Procedure P1 (Most-violated variable below its minimum)

Given constraint row R, whose basis variable is below its minimum and is the worst violation. Pick column S, so that, $c_S/(-a_{R,S})$ is minimum for all S that meet the following rules:

a. S is not in the current basis.
b. $a_{R,S}$ is not equal to zero.
c. If x_S is at its minimum, then $a_{R,S}$ must be negative and c_S must be positive or zero.
d. If x_S is at its maximum, then $a_{R,S}$ must be positive and c_S must be negative or zero.

Column Selection Procedure P2 (Most-violated variable above its maximum)

Given constraint row R, whose basis variable is above its maximum and is the worst violation. Pick column S, so that, $c_S/a_{R,S}$ is the minimum for all S that meet the following rules:

a. S is not in the current basis.
b. $a_{R,S}$ is not already zero.

c. If x_S is at its minimum, then $a_{R,S}$ must be positive and c_S must be positive or zero.
d. If x_S is at its maximum, then $a_{R,S}$ must be negative and c_S must be negative or zero.

3. When a column has been selected, pivot at the selected row R (from step 1) and column S (from step 2). The pivot column's variable, S, goes into the basis.

If no column fits the column selection criteria, we have an infeasible solution. That is, there are no values for $x_1 \dots x_N$ that will satisfy all constraints

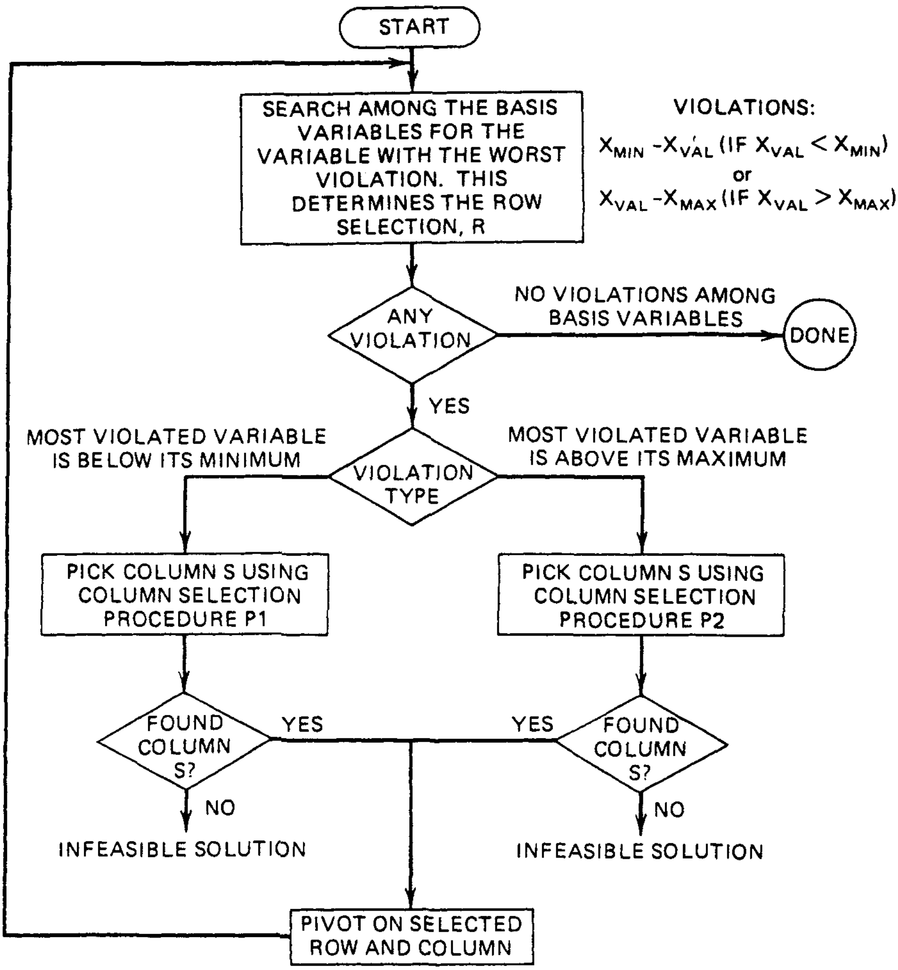


FIG. 6.10 Dual upper-bound linear programming algorithm.

simultaneously. In some problems, the cost coefficient c_S associated with column S will be zero for several different values of S. In such a case, $c_S/a_{R,S}$ will be zero for each such S and none of them will be the minimum. The fact that c_S is zero means that there will be no increase in cost if any of the S values are pivoted into the basis; therefore, the algorithm is indifferent to which one is chosen.

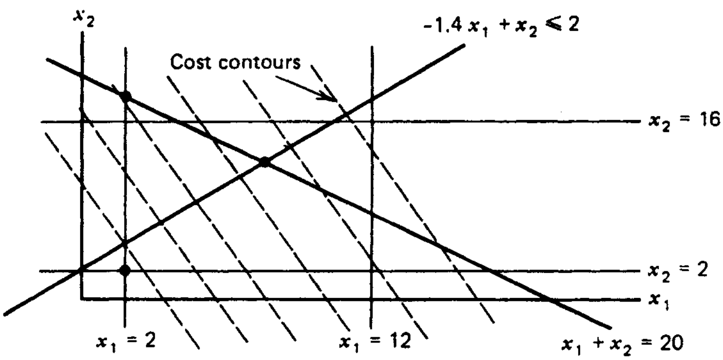
Setting the Variables after Pivoting

1. All nonbasis variables, except x_S, remain as they were before pivoting.
2. The most violated variable is set to the limit that was violated.
3. Since all nonbasis variables are determined, we can proceed to set each basis variable to whatever value is required to make the constraints balance. Note that this last step may move all the basis variables to new values, and some may now end up violating their respective limits (including the x_S variable).

Go back to step 1 of the variable exchange procedure.

These steps are shown in flowchart form in Figure 6.10. To help you understand the procedures involved, a sample problem is solved using the dual upper-bounding algorithm. The sample problem, shown in Figure 6.11, consists of a two-variable objective with one equality constraint and one inequality constraint.

First, we must put the equations into canonical form by adding slack variables x_3 and x_4. These variables are given limits corresponding to the type of constraint into which they are placed, x_3 is the slack variable in the equality



Minimize: $Z = 2x_1 + x_2$

Subject to: $x_1 + x_2 = 20$ constraint 1

$-1.4x_1 + x_2 \leq 2$ constraint 2

$2 \leq x_1 \leq 12$

$2 \leq x_2 \leq 16$

FIG. 6.11 Sample linear programming problem.

constraint, so its limits are both zero; x_4 is in an inequality constraint, so it is restricted to be a positive number. To start the problem, the objective function must be set to the minimum value it can attain, and the algorithm will then seek the minimum constrained solution by increasing the objective just enough to reach the constrained solution. Thus, we set both x_1 and x_2 at their minimum values since the cost coefficients are both positive. These conditions are shown here:

Constraint 1:	x_1 +	x_2	$+ x_3$		$= 20 \leftarrow$ R
Constraint 2:	$-1.4x_1$ +	x_2		$+ x_4$	$= 2$
Cost:	$2x_1$ +	x_2			$-Z = 0$
					$0 \leq x_3 \leq 0$
					$0 \leq x_4 \leq \infty$
Minimum:	2	2	0	0	
Present value:	2	2	16	2.8	6
Maximum:	12	16	0	∞	
			Basis variable 1 ↑ Worst-violated variable	Basis variable 2	

We can see from these conditions that variable x_3 is the worst-violated variable and that it presently exceeds its maximum limit of zero. Thus, we must use column procedure P2 on constraint number 1. This is summarized as follows:

Using selection procedure P2 on constraint 1:

$i = 1 \quad a_1 > 0 \quad x_1 = x_1^{\min} \quad c_1 > 0 \quad \text{then} \quad \frac{c_1}{a_1} = \frac{2}{1} = 2$

$i = 2 \quad a_2 > 0 \quad x_2 = x_2^{\min} \quad c_1 > 0 \quad \text{then} \quad \frac{c_2}{a_2} = \frac{1}{1} = 1$

min c_i/a_i is 1 at $i = 2$

Pivot at column 2, row 1

To carry out the required pivot operations on column 2, row 1, we need merely subtract the first constraint from the second constraint and from the objective function. This results in:

Constraint 1:	x_1	$+ x_2 + x_3$				$= 20$
Constraint 2:	$-2.4x_1$	$- x_3$		$+ x_4$		$= -18 \leftarrow$ R
Cost:	x_1	$- x_3$				$-Z = -20$
Minimum:	2	2	0	0		
Present value:	2	18	0	-13.2	22	
Maximum:	12	16	0	∞		
		Basis variable 1		Basis variable 2 ↖ Worst-violated variable		

We can see now that the variable with the worst violation is x_4 and that x_4 is below its minimum. Thus, we must use selection procedure P1 as follows:

Using selection procedure P1 on constraint 2:

$i = 1 \quad a_1 < 0 \quad x_1 = x_1^{\min} \qquad c_1 > 0 \quad \text{then } \dfrac{c_1}{-a_1} = \dfrac{1}{-(-2.4)} = 0.4166$

$i = 3 \quad a_3 < 0 \quad x_3 = x_3^{\min} = x_3^{\max} \; c_3 < 0 \quad \text{then } x_3 \text{ is not eligible}$

Pivot at column 1, row 2

After pivoting, this results in:

Constraint 1:		x_2	$+ 0.5833x_3$	$+ 0.4166x_4$		$= 12.5$
Constraint 2:	x_1		$+ 0.4166x_3$	$- 0.4166x_4$		$= 7.5$
Cost:			$- 1.4166x_3$	$+ 0.4166x_4$	$- Z$	$= -27.5$
Minimum:	2	2	0	0		
Present value:	7.5	12.5	0	0	−27.5	
Maximum:	12	16	0	∞		
	Basis variable 1	Basis variable 2				

At this point, we have no violations among the basis variables, so the algorithm can stop at the optimum.

$$\left.\begin{array}{l} x_1 = 7.5 \\ x_2 = 12.5 \end{array}\right\} \text{cost} = 27.5$$

See Figure 6.11 to verify that this is the optimum. The dots in Figure 6.11 show the solution points beginning at the starting point $x_1 = 2$, $x_2 = 2$, cost = 6.0, then going to $x_1 = 2$, $x_2 = 18$, cost = 22.0, and finally to the optimum $x_1 = 7.5$, $x_2 = 12.5$, cost = 27.5.

How does this algorithm work? At each step, two decisions are made.

1. Select the most-violated variable.
2. Select a variable to enter the basis.

The first decision will allow the procedure to eliminate, one after the other, those constraint violations that exist at the start, as well as those that happen during the variable-exchange steps. The second decision (using the column selection procedures) guarantees that the rate of increase in cost, to move the violated variable to its limit, is minimized. Thus, the algorithm starts from a minimum cost, infeasible solution (constraints violated), toward a minimum cost, feasible solution, by minimizing the rate of cost increase at each step.

PROBLEMS

6.1 Three units are on-line all 720 h of a 30-day month. Their characteristics are as follows:

$$H_1 = 225 + 8.47P_1 + 0.0025P_1^2,\ 50 \le P_1 \le 350$$

$$H_2 = 729 + 6.20P_2 + 0.0081P_2^2,\ 50 \le P_2 \le 350$$

$$H_3 = 400 + 7.20P_3 + 0.0025P_3^2,\ 50 \le P_3 \le 450$$

In these equations, the H_i are in MBtu/h and the P_i are in MW.

Fuel costs for units 2 and 3 are 0.60 ₨/MBtu. Unit 1, however, is operated under a take-or-pay fuel contract where 60,000 tons of coal are to be burned and/or paid for in each 30-day period. This coal costs 12 ₨/ton delivered and has an average heat content of 12,500 Btu/lb (1 ton = 2000 lb).

The system monthly load-duration curve may be approximated by three steps as follows.

Load (MW)	Duration (h)	Energy (MWh)
800	50	40000
500	550	275000
300	120	36000
Total	720	351000

a. Compute the economic schedule for the month assuming all three units are on-line all the time and that the coal must be consumed. Show the MW loading for each load period, the MWh of each unit, and the value of gamma (the pseudo-fuel cost).

b. What would be the schedule if unit 1 was burning the coal at 12 ₨/ton with no constraint to use 60,000 tons? Assume the coal may be purchased on the spot market for that price and compute all the data asked for in part a. In addition, calculate the amount of coal required for the unit.

6.2 Refer to Example 6A, where three generating units are combined into a single composite generating unit. Repeat the example, except develop an equivalent incremental cost characteristic using only the incremental characteristics of the three units. Using this composite incremental characteristic plus the zero-load intercept costs of the three units, develop the total cost characteristic of the composite. (Suggestion: Fit the composite incremental cost data points using a linear approximation and a least-squares fitting algorithm.)

6.3 Refer to Problem 3.8, where three generator units have input–output curves specified as a series of straight-line segments. Can you develop a composite input–output curve for the three units? Assume all three units are on-line and that the composite input–output curve has as many linear segments as needed.

6.4 Refer to Example 6E. The first problem solved in Example 6E left the end-point restrictions at zero to 200,000 tons for both coal piles at the end of the 3-wk period. Resolve the first problem [$V_1(1) = 70{,}000$ and $V_2(1) = 70{,}000$] with the added restriction that the final volume of coal at plant 2 at the end of the third week be at least 20,000 tons.

6.5 Refer to Example 6E. In the second case solved with the LP algorithm (starting volumes equal to 70,000 and 50,000 for plant 1 and plant 2, respectively), we restricted the final volume of the coal pile at plant 2 to be 8000 tons. What is the optimum schedule if this final volume restriction is relaxed (i.e., the final coal pile at plant 2 could go to zero)?

6.6 Using the linear programming problem in the text shown in Example 6E, run a linear program to find the following:

1. The coal unloading machinery at plant 2 is going to be taken out for maintenance for one week. During the maintenance work, no coal can be delivered to plant 2. The plant management would like to know if this should be done in week 2 or week 3. The decision will be based on the overall three-week total cost for running both plants.
2. Could the maintenance be done in week 1? If not, why not?

Use as initial conditions those found in the beginning of the sample LP executions found in the text; i.e., $V_1(1) = 70{,}000$ and $V_2(2) = 70{,}000$.

6.7 The "Cut and Shred Paper Company" of northern Minnesota has two power plants. One burns coal and the other burns natural gas supplied by the Texas Gas Company from a pipeline. The paper company has ample supplies of coal from a mine in North Dakota and it purchases gas as take-or-pay contracts for fixed periods of time. For the 8-h time period shown below, the paper company must burn $15 \cdot 10^6$ ft^3 of gas.

The fuel costs to the paper company are

Coal:	0.60 \$/MBtu
Gas:	2.0 \$/ccf (where 1 ccf = 1000 ft^3) the gas is rated at 1100 Btu/ft^3

Input–output characteristics of generators:

Unit 1 (coal unit): $H_1(P_1) = 200 + 8.5P_1 + 0.002P_1^2$ MBtu/h

$50 < P_1 < 500$

Unit 2 (gas unit): $H_2(P_2) = 300 + 6.0P_2 + 0.0025P_2^2$ MBtu/h

$50 < P_2 < 400$

Load (both load periods are 4 h long):

Period	Load (MW)
1	400
2	650

Assume both units are on-line for the entire 8 h.

Find the most economic operation of the paper company power plants, over the 8 h, which meets the gas consumption requirements.

6.8 Repeat the example in the Appendix, replacing the $x_1 + x_2 = 20$ constraint with:

$$x_1 + x_2 < 20$$

Redraw Figure 6.11 and show the admissible, convex region.

6.9 An oil-fired power plant (Figure 6.12) has the following fuel consumption curve.

$$q(\text{bbl/h}) = \begin{cases} 50 + P + 0.005P^2 & \text{for } 100 \leq P \leq 500 \text{ MW} \\ 0 & \text{for } P = 0 \end{cases}$$

The plant is connected to an oil storage tank with a maximum capacity of 4000 bbl. The tank has an initial volume of oil of 3000 bbl. In addition, there is a pipeline supplying oil to the plant. The pipeline terminates in the same storage tank and must be operated by contract at 500 bbl/h. The oil-fired power plant supplies energy into a system, along with other units. The other units have an equivalent cost curve of

$$F_{eq} = 300 + 6P_{eq} + 0.0025P_{eq}^2$$

$$50 \leq P_{eq} \leq 700 \text{ MW}$$

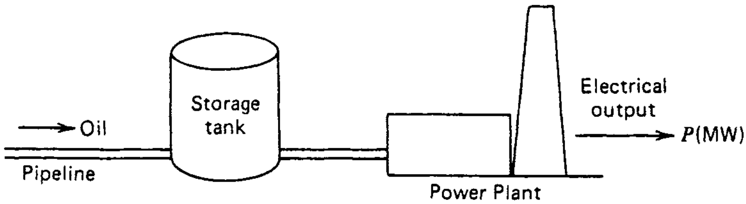


FIG. 6.12 Oil-fired power plant with storage tank for Problem 6.9.

The load to be supplied is given as follows:

Period	Load (MW)
1	400
2	900
3	700

Each time period is 2 h in length. Find the oil-fired plant's schedule using dynamic programming, such that the operating cost on the equivalent plant is minimized and the final volume in the storage tank is 2000 bbl at the end of the third period. When solving, you may use 2000, 3000, and 4000 bbl as the storage volume states for the tank. The q versus P function values you will need are included in the following table.

q(bbl/h)	P(MW)
0	0
200	100.0
250	123.6
500	216.2
750	287.3
1000	347.2
1250	400.0
1500	447.7
1800	500.0

The plant may be shut down for any of the 2-h periods with no start-up or shut-down costs.

FURTHER READING

There has not been a great deal of research work on fuel scheduling as specifically applied to power systems. However, the fuel-scheduling problem for power systems is not really that much different from other "scheduling" problems, and, for this type of problem, a great deal of literature exists.

References 1–4 are representative of efforts in applying scheduling techniques to the power system fuel-scheduling problem. References 5–8 are textbooks on linear programming that the authors have used. There are many more texts that cover LP and its variations. The reader is encouraged to study LP independently of this text if a great deal of use is to be made of LP. Many computing equipment and independent software companies have excellent LP codes that can be used, rather than writing one's own code. Reference 8 is the basis for the algorithm in the appendix to this chapter. References 9–11 give recent techniques used.

1. Trefny, F. J., Lee, K. Y., "Economic Fuel Dispatch," *IEEE Transactions on Power Apparatus and Systems*, Vol. 100, July 1981, 3468–3477.
2. Seymore, G. F., "Fuel Scheduling for Electric Power Systems," in A. M. Erisman, K. W. Noves, M. H. Dwarakanath (eds.), *Electric Power Problems: The Mathematical Challenge*, SIAM, Philadelphia, 1980, pp. 378–392.
3. Lamont, J. W., Lesso, W. G. "An Approach to Daily Fossil Fuel Management," in A. M. Erisman, K. W. Noves, M. H. Dwarakanath (eds), *Electric Power Problems: The Mathematical Challenge*, SIAM, Philadelphia, 1980, pp. 414–425.
4. Lamont, J. W., Lesso, W. G., Rantz, M., "Daily Fossil Fuel Management," 1979 *PICA Conference Proceedings*, IEEE Publication, 79CH1381-3-PWR, pp. 228–235.
5. Lasdon, L. S., *Optimization Theoryfor Large Systems*, Macmillan, New York, 1970.
6. Hadley, G., *Linear Programming*, Addison-Wesley, Reading, MA, 1962.
7. Wagner, H. M., *Principle of Operations Research with Application to Managerial Decisions*, Prentice-Hall, Englewood Cliffs, NJ, 1975.
8. Wagner, H. M., "The Dual Simplex Algorithm for Bounded Variables," *Naval Research Logistics Quarterly*, Vol. 5, 1958, pp. 257–261.
9. Rosenberg, L. D., Williams, D. A., Campbell, J. D., "Fuel Scheduling and Accounting," *IEEE Transactions on Power Systems*, Vol. 5, No. 2, May 1990, pp. 682–688.
10. Lee, F. N., "Adaptive Fuel Allocation Approach to Generation Dispatch using Pseudo Fuel Prices," *IEEE Transactions on Power Systems*, Vol. 7, No. 2, May 1992, pp. 487–496.
11. Sherkat, V. R., Ikura, Y., "Experience with Interior Point Optimization Software for a Fuel Planning Application," 1993 *IEEE Power Industry Computer Applications Conference*, pp. 89–96.