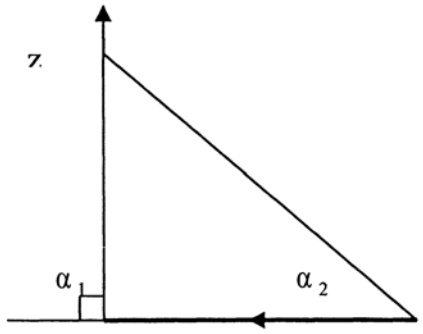
# Chapter 6—Practice Examples

## P.E. 6.1




$$\rho = 5,\ \cos\alpha_1 = 0,\quad \cos\alpha_2 = \sqrt{\frac{2}{27}}$$

$$\mathrm{a}_\phi = \mathrm{a}_1 \times \mathrm{a}_\rho = \left(\frac{-a_x - a_y}{\sqrt{2}}\right) \times a_z = \frac{-a_x + a_y}{\sqrt{2}}$$

$$H_3 = \frac{10}{4\pi(5)}\left(\sqrt{\frac{2}{27}} - 0\right)\left(\frac{-a_x + a_y}{\sqrt{2}}\right) = \underline{\underline{-30.63a_x + 30.63a_y}}\ \text{mA/m}$$

## P.E. 6.2

(a) $$\boldsymbol{H} = \frac{2}{4\pi(2)}\left(1 + \frac{3}{\sqrt{13}}\right)\boldsymbol{a}_z = \underline{\underline{0.1458\boldsymbol{a}_z}}\ \text{A/m}$$

(b) $$\rho = \sqrt{3^2 + 4^2} = 5, \alpha_2 = 0, \cos\alpha_1 = -\frac{12}{13},$$

$$\mathrm{a}_\phi = -\mathrm{a}_y \times \left(\frac{3\mathrm{a}_x - 4\mathrm{a}_z}{5}\right) = \frac{4\mathrm{a}_x + 3\mathrm{a}_z}{5}$$

$$\boldsymbol{H} = \frac{2}{4\pi(5)}\left(1 + \frac{12}{13}\right)\left(\frac{4\mathrm{a}_x + 3\mathrm{a}_z}{5}\right) = \frac{1}{26\pi}\left(4\mathrm{a}_x + 3\mathrm{a}_z\right)$$

$$= \underline{\underline{48.97a_x + 36.73a_z}}\ \text{mA/m}$$

**P.E. 6.3**

(a) From Examples 6.3

$$H = \frac{Ia^2}{2(a^2 + z^2)^{3/2}} a_z$$

At (0,0,-1cm), z = 2cm,

$$H = \frac{50 \times 10^{-3} \times 25 \times 10^{-4}}{2(5^2 + 2^2)^{3/2} \times 10^{-6}} a_z \quad \text{A/m}$$

$$= \underline{\underline{400.2a_z \ \text{mA/m}}}$$

(b) At (0,0,10cm), z = 9cm,

$$\boldsymbol{H} = \frac{50 \times 10^{-3} \times 25 \times 10^{-4}}{2(5^2 + 9^2)^{3/2} \times 10^{-6}} a_z$$

$$= \underline{\underline{57.3a_z \ \text{mA/m}}}$$

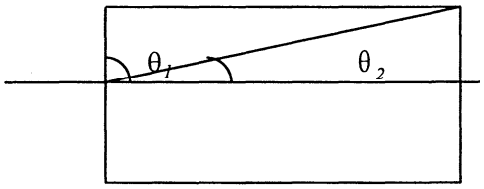
## P.E. 6.4

$$H = \frac{NI}{2L}\left(\cos\theta_2 - \cos\theta_1\right)a_z = \frac{2\times 10^3 \times 50 \times 10^{-3}(\cos\theta_2 - \cos\theta_1)a_z}{2\times 0.75}$$

$$= \frac{100}{1.5}\left(\cos\theta_2 - \cos\theta_1\right)\boldsymbol{a}_z$$

(a) At (0,0,0), $\theta = 90^o$, $\cos\theta_2 = \dfrac{0.75}{\sqrt{0.75^2 + 0.05^2}}$

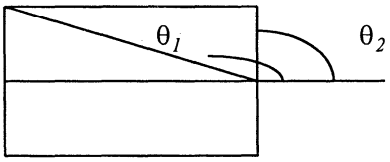
$= 0.9978$




$$H = \frac{100}{1.5}(0.9978 - 0)a_z$$

$= \underline{66.52\ \mathbf{a}_z\ \text{A/m}}$

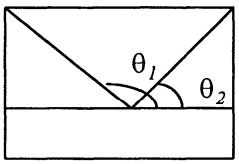
(b) At (0,0,0.75), $\theta_2 = 90^o$, $\cos\theta_1 = -0.9978$




$$\boldsymbol{H} = \frac{100}{1.5}(0 + 0.9978)\boldsymbol{a}_z$$

$= \underline{66.52\mathbf{a}_z\ \text{A/m}}$

(c) At (0,0,0.5), $\cos\theta_1 = \dfrac{-0.5}{\sqrt{0.5^2 + 0.05^2}} = -0.995$




$$\cos\theta_1 = \frac{0.25}{\sqrt{0.25^2 + 0.05^2}} = 0.9806$$

$$\boldsymbol{H} = \frac{100}{1.5}(0.9806 + 0.995)\boldsymbol{a}_z$$

$= \underline{131.7\mathbf{a}_z\ \text{A/m}}$

## P.E. 6.5

$$\boldsymbol{H} = \frac{1}{2}\boldsymbol{K} \times \boldsymbol{a}_n$$

(a) $H(0,0,0) = \dfrac{1}{2}50a_z \times (-a_y) = \underline{25a_x}$ mA/m

(b) $H(1,5,-3) = \dfrac{1}{2}50a_z \times a_y = \underline{-25a_x}$ mA/m

## P.E. 6.6

$$|H| = \begin{cases} \dfrac{NI}{2\pi\rho}, & \rho - a < \rho < \rho + a, \quad 9 < \rho < 11 \\ 0, & \text{otherwise} \end{cases}$$

(a) At $(3,-4,0)$, $\rho = \sqrt{3^2 + 4^2} = 5\text{cm} < 9\text{cm}$

$$|H| = \underline{\underline{0}}$$

(b) At $(6,9,0)$, $\rho = \sqrt{6^2 + 9^2} = \sqrt{117} < 11$

$$|H| = \frac{10^3 \times 100 \times 10^{-3}}{2\pi\sqrt{117} \times 10^2} = \underline{\underline{147.1}} \text{ A/m}$$

## P.E. 6.7

(a) $$\boldsymbol{B} = \nabla \times \boldsymbol{A} = (-4xz - 0)\boldsymbol{a}_x + (0 + 4yz)\boldsymbol{a}_y + (y^2 - x^2)\boldsymbol{a}_z$$

$$B(-1,2,5) = \underline{\underline{20a_x + 40a_y + 3a_z}} \text{ Wb/m}^2$$

(b) $$\psi = \int B.\partial s = \int_{y=-1}^{4}\int_{x=0}^{1} (y^2 - x^2)\partial x \partial y = \int_{-1}^{4} y^2 \partial y - 5\int_{0}^{1} x^2 \partial x$$

$$= \frac{1}{3}(64 + 1) - \frac{5}{3} = \underline{\underline{20}} \text{ Wb}$$

Alternatively,

$$\psi = \int \mathbf{A}.\partial \mathbf{l} = \int_{0}^{1} \mathbf{x}^2(-1)\partial \mathbf{x} + \int_{-1}^{4} \mathbf{y}^2(1)\partial \mathbf{y} + \int_{1}^{0} \mathbf{x}^2(4)\partial \mathbf{x} + 0$$

$$= -\frac{5}{3} + \frac{65}{3} = \underline{\underline{20}} \text{ Wb}$$